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Special Issue

# Times, Events, and Logical Specification

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Guest Editors

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# Preface

The current special issue of the *Organon F* journal is for all interested in the history, present, and future challenges of temporal logic or in the attempts to analyze temporal phenomena or time-dependent entities from the perspective of logic, philosophy, or analytic metaphysics. This issue is primarily based on the accepted and reviewed contributions from the conference *Times, Events, and Logical Specification* held on May 19, 2022, till May 21, 2022, at the Department of Philosophy, Palacký University Olomouc.

We would like, first of all, to thank all the authors for their valuable and inspiring contributions presented at the conference or in this current issue. At the same time, we are grateful to all the reviewers of the papers for their careful work, and precious and helpful comments.

As the title of the conference suggests, the main focus was on the phenomenon of time and events, their logical specification, and associated ontological commitments. The problem of explicitly capturing ongoing processes and temporality is becoming increasingly widespread not only in analytical philosophy and logic but also in computer science. Some papers dealt with the issue from a historical perspective; others presented new challenges and solutions.

Peter Øhrstrøm and David Jakobsen devote their papers on the history of tense logic, focusing specifically on the logic of Arthur Prior. Peter Øhrstrøm suggests in his paper *Highlights in the Development of Tense-Logic* that Prior's personal motivation consists of the tension between human free will and divine foreknowledge. He points out that Prior developed his tense logic to deal with this issue. Despite Prior's motivation being well known, Øhrstrøm's paper provides new archival material that complements the history of the rise of tense logic.

Jakobsen's article *Fulfilling Russell's Wish: A.N. Prior and the Resurgence of Philosophical Theology* introduced Prior's contribution to the discussions on ontological argument and analytic theology. Similarly

to Øhrstrøm's paper, Jakobsen presented unpublished archival materials that illustrate Prior's interest in the issue and the struggles he had to undergo in contemporary analytic discourse. Additionally, Jakobsen develops Prior's argument of metaphysical and theological issues that quantified tense-logic has brought.

Since changes taking place over time are usually called events, the specification of the term event and ontological commitments connected to it becomes crucial in the context of temporal phenomena. Bjørn Jespersen and Massimiliano Carrara open the issue of the knowability of impossible events in their article *Impossible events and the knowability paradox*. Their contribution answers the disturbing question of whether impossible events are unknowable. It begins by distinguishing between concepts of 'impossible knowledge' and 'knowledge of impossibility'. Their analytical tool to rigorously provide their answer is Transparent Intensional Logic (TIL).

The same analytical tool as the specification language is used by Marie Duží when analyzing the events and activities of agents in multi-agent systems in her article *Specification of Agents' Activities in Past, Present and Future*. She demonstrates the importance of the fine-grained analysis of agents' activities and points out that the relevance of events to agents might differ over time. She provides the analysis of dynamic activities in TIL with respect to the present, past, and future time.

Tadeusz Ciecierski argues in his paper *Actions, Products, Demonstrations* that the concept of demonstrations contains the ambiguity between actions and products and provides a theory of demonstrations based on this distinction. He pointed out that when one handles demonstrative situations the proposed theory offers a rich explanatory and descriptive power.

This issue does not include contributions from conference organizers that were also presented at the conference *Times, Events, and Logical Specification*. Martina Číhalová's paper *Specification of the Fundamental Concepts in the Ontology of Processes; Event, Process, Activity* and Zuzana Rybaříková's paper *Prior and Tichý's Concepts of Temporalism* were published in the previous issue of *Organon F*.

As conferences tend to be a succession of lesser or greater disasters, which bring sleepless nights to organizers, we prepared to quote one of the titles of Prior's papers, *Thank Goodness That's Over*, at the end of our conference. However, we did not use it in the end. Although our meeting started with a broken coffee machine

and some other sequence of troubles happened as is common at such events, the contributions and discussions were so impressive that it was a great time for us and we could even easily survive a longer duration. Our sincere thanks to everyone involved!

We hope that the papers included in this special issue will have a similar impact on you, readers, and will motivate you to read more impressive texts from our authors or at least about the presented topics.

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RESEARCH ARTICLE

# Highlights in the Development of Tense-Logic

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*Abstract*. In 1954, on the 27th August, Arthur Norman Prior presented his ideas of tense-logic[1] for the first time. He developed the field further in many publications until his death in 1969. His books *Time and Modality* (1957a), *Past, Present and Future* (1967), and *Papers on Time and Tense* (1968) were clearly vital milestones. Much of Prior's personal motivation had to do with his struggle with the logical tension between the theological doctrines of divine foreknowledge and human freedom. It turned out that tense-logic gave rise to a powerful tool for dealing with this and similar problems. Furthermore, important highlights in Prior's tense-logic were the development of branching time and the introduction of instant propositions (leading to what has later been called 'hybrid logic'). Since Prior's death, many further developments of formal tense-logic and its semantics have been presented and carefully investigated. In philosophical logic, many researchers have focused on discussions regarding 'the true future' and the notion of 'the thin red line'.

*Keywords*: Tense-logic; A.N. Prior; time; modality; determinism.

[1] The hyphenated term 'tense-logic' and the nonhyphenated 'tense logic' are both used in the literature. In this paper the hyphenated term will be chosen as this is what Prior did in his important books, *Time and Modality* (1957a) and *Past, Present and Future* (1967).

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## 1. The beginning of tense-logic

Tense-logic is one of the most important contributions to logic during the 20th century. It offers a new formalism in which tenses (past, future, etc.) are represented as propositional operators ($P$, $F$, etc.). Tense-logic also reintroduced into modern logic the ancient idea that the truth value of a proposition can change from time to time.

The founding father of tense-logic is Arthur Norman Prior (1914–69). He presented his basic ideas of tense-logic for the very first time at a conference in Wellington, New Zealand, in 1954 (see [Prior 1958]), and within a few years, this new development of logic became known by logicians all over the world. However, it should be mentioned that there were other scholars who had played important roles in the work that led to tense-logic. One of them was Henrik von Wright (1916–2003), who had been a great inspiration to Prior. In fact, Prior explicitly mentioned von Wright's important work on the logic of futurity in his famous lecture in Wellington in 1954. Even more important was probably Prior's philosophical and theological interests in fundamental and existential questions regarding determinism and human freedom. As we shall see, his struggle with such problems during the 1930s and 1940s apparently led him to look for a new logic of time.

## 2. Prior's motivation for the struggle toward a new logical framework for the study of time

Much of Prior's personal motivation for working with problems regarding time had to do with his study of the logical tension between the Christian doctrines of divine foreknowledge and human freedom.

Prior's interest in the problem of determinism and its philosophical and theological aspects can be traced back to his early years. Already, as a teenager, he rejected the Methodism of his parents and became a Calvinist. Actually, in 1931, when he was only 17 years old, he wrote some rather detailed essays on problems related to determinism and time (see [Jakobsen et al. 2021]). He found Bergson's arguments against Einstein's space-time

unconvincing and, having accepted Einstein's ideas, he found that everything that has happened and everything that is going to happen must be accessible from God's perspective. He concluded that this means that there cannot be any human freedom of choice.

During the 1930s and 1940s, Prior carried out numerous Calvinistic studies, and he became a very active member of the Presbyterian Church, although he also had periods of doubt, particularly after 1940. He emphasized that there is a long tradition in theology of rejecting the doctrine of free will:

> ... a whole line of Christian thinkers, running from Augustine (to trace it back no further) through Luther and Calvin and Pascal to Barth and Brunner in our own day, have attacked freewill in the name of religion.
> … Jonathan Edwards, the 18th–century New England divine who produced a novel defence of Calvinism … simply demonstrating the absurdity of freewill itself… (Prior 2022a, 1)

During the 1940s Prior gradually changed his view on human freedom and finally he became a defender of free choice. Around 1950, when Prior worked as a senior lecturer at Canterbury University College, Christchurch, he was apparently looking for a logical framework that would be useful for the further studies of the relations between the doctrines of human freedom and divine foreknowledge. Prior accepted that there are future contingents, i.e., propositions about the future that are neither necessary nor impossible. But what can be said about the truth values of such propositions? Clearly, a believer in complete divine foreknowledge will have to say that God knows the truth values of all future contingents. Prior found that we need a precise formalism to explore the logical possibilities if we want to hold on to the doctrine of human freedom, along with the belief in God's complete foreknowledge. After intensely researching modal logic in the following few years, Prior realised that to analyse such problems, tenses would have to be taken seriously in logic; that is, we must include the relevant tenses in the formalism we are using to carry out the logical analysis.

In 1951, Prior became an elder of the Presbyterian Church in Christchurch. The same year, he also attended the Philosophical Congress in Sydney, Australia, where he made important friendships with J.L. Mackie and

J.J.C. Smart and other philosophers who were interested in topics related to logic and time.

In 1952, Prior was appointed as professor of philosophy at Canterbury University College, Christchurch. In 1953, he organised the first national conference on philosophy in New Zealand, and he became president of the *New Zealand Section of the Australasian Association of Psychology and Philosophy.*

In 1953, Prior also published a paper dealing with the problem of future contingency in terms of a three-valued logic (1953). For a few years, Prior thought that the use of a third truth value 'was the only way to present an indeterminist tense-logic' (1967, 128-29). However, later he was able to show that there are interesting alternatives (1957a, 94 ff.). As he had realised that a three-valued logic could give rise to complications, he found that it would be better to stick to a traditional, bivalent logic.

## 3. Toward a tense-logical formalism

In the beginning of June 1954, Prior was preparing his presidential address, which he was supposed to give at *The Second Philosophical Congress organized by New Zealand Section of the Australasian Association of Psychology and Philosophy*, Wellington 27–30 August 1954. On 6 June 1954, he wrote a letter to his wife, Mary, who was then in hospital. In this letter, Prior explained that he intended to present 'a long thing on "The Syntax of Time Distinctions", which is going to be a classic'. (Prior 2022b) It is obvious that he had very high expectations of this presentation of his ideas on time and logic. His hope was apparently that this would mark the beginning of a new approach to the study of logic and time. He even indicated something about the next steps in this work. He wrote, 'Later on I may work on interaction between tense-logic and deontic logic, but that's way up in the air at present.' (Prior 1954).

Prior had got important inspiration from reading a footnote by John Findlay, and he told Mary about it when he visited her (probably at the hospital):

> … he came and sat on the bed in high excitement. He read the all important footnote. He felt he could formalise tense distinctions, drawing inspiration from this footnote of Findlay's. (Interview with Mary Prior, included in [Prior 2003, 297])

The footnote in question is as follows:

> And our conventions with regard to tenses are so well worked out that we have practically the materials in them for a formal calculus... The calculus of tenses should have been included in the modern development of modal logics. It includes such obvious propositions as that
>
> x present = (x present) present
>
> x future = (x future) present = (x present) future;
>
> also such comparatively recondite propositions as that
>
> (x).(x past)future; i.e. all events, past and future will be past. (Findlay 1941, 233)

Early in 1954 Prior had studied Benson Mates book, *Stoic Logic* (1953). In particular, he was interested in the Master Argument which Mates presented in his book in the following manner:

> Diodorus argued that the following three propositions could not all be true.
>
> (1) Every proposition true about the past is necessary.
>
> (2) An impossible proposition may not follow from a possible one.
>
> (3) There is a proposition which is possible, but which neither is true nor will be true.
>
> Since, according to Epictetus, the first two propositions seemed to Diodorus to be more plausible than the third, he dropped the third, and this accounts for his definition of the possible as 'that which either is true or will be true'. (Mates 1953, 38)

According to Mates (1953), the details of the Diodorean Master Argument are not known. However, Prior was eager construct an argument by which it can be demonstrated that the denial of (3) follows from (1) and (2) in the Diodorean argument. In fact, he managed to do so using a formal language like the one suggested by Findlay. His proof was written in early 1954 and published in (1955). It shows that given (1) and (2) the possible is 'that

which either is true or will be true'. Prior referred to this Diodorean concept of possibility in his presidential address in August 1954 (1958a, 110), and in the following years he often returned to the Master Argument and similar arguments. As we shall see in section 5, he even discussed the Diodorean argument in his most important book, *Past, Present, and Future* (1967). There can be no doubt that what Prior learned from Mates was one of the important highlights in the development of tense-logic. In a letter to Mates dated 6 August 1954, Prior wrote: 'It goes without saying that I've enjoyed & profited by your book immensely' (2022e).

Prior admitted that the formalisation of the use of tenses may be a very complicated project. In his presidential address, he quoted C.S. Peirce, who in 1903, had stated:

> Time has usually been considered by logicians to be what is called 'extra-logical' matter. I have never shared this opinion. But I have thought that logic had not yet reached the state of development at which the introduction of temporal modifications of its forms would not result in great confusion; and I am much of that way of thinking yet. (Peirce 1931, 4.523)

However, Prior maintained that it would now be possible to carry out what Peirce hesitated to do in 1903:

> What the time was not ripe for in 1903, it may well be ripe for now, for in the intervening period, we have acquired a vast fund of knowledge about the possible structures of modal systems, and (as the scholastic logicians knew) tense and mood are species of the same genus. (Prior 1958a, 106)

Prior apparently found that one of the things that had made the formulation of tense-logic possible was a deeper understanding of what 'the scholastic logicians knew'. In fact, there is a strong emphasis on the importance of wisdom formulated in scholastic logic, mainly in the logic developed by William of Ockham (c. 1287–1347), who wrote a book that significantly inspired Prior (William of Ockham 1945). According to Ockham (and medieval logic in general) logic should include the study of propositions 'in the ancient and medieval sense' (Prior 1958a, 105 & 113), i.e., propositions that may change their truth-values relative to the time of uttering. Furthermore,

Ockham (1945) had demonstrated the importance of dealing not only with propositions in the present tense, but also with propositions in the past and the future tenses. With his tense-logic, Prior wanted to formulate this medieval wisdom in terms of modern symbolic logic.

Prior's presidential address in Wellington in 1954 marked the beginning of the worldwide development of tense-logic. The ideas presented in this lecture quickly became known among important logicians in several countries, and Prior himself was very active in the further development of the new field. In particular, it was of great importance that Prior was invited to deliver the John Locke Lectures for 1955–56 at the University of Oxford. These lectures led to the publication of the first book on the topic of time and modality (1957a). The publication of the Wellington lecture, the presidential address from 27 August 1954, had to wait until 1958, when it was published in *Franciscan Studies* (1958a). Given Prior's emphasis on Ockham's logic in the lecture, it was natural to submit it to this journal. After all, William of Ockham was a Franciscan friar.

## 4. Prior's approach to tense-logic in his presidential address, 27 August, 1954

In his famous Wellington lecture held in 1954 (1958a), Prior referred to Henrik von Wright's modal system, which may be understood as an extension of propositional logic with an operator, $M$ (resp. 'It is possible that'), that obeys the following axioms:

B1: $p \supset Mp$

B2: $M(p \vee q) \equiv (Mp \vee Mq)$

and the rules

RB1: $\alpha \equiv \beta \rightarrow L\alpha \equiv M\beta$

RB2: $\alpha \rightarrow L\alpha$, where $L = {\sim}M{\sim}$

From this modal logic, the logic of futurity is obtained by excluding B1 and interpreting $Mp$ as

*F:* 'it will be the case that …'

and assuming

F3: $FFp \equiv Fp$

Similarly, Prior obtained a logic of pastness from von Wright's modal system by interpreting $Mp$ as

*P:* 'it has been the case that …'

and by assuming

P3: $PPp \equiv Pp$

Furthermore, Prior introduced two additional tense-logical operators:

*G:* 'it will always be the case that …'

*H:* 'it has always been the case that …'

defined as $G \equiv \sim F\sim$ and $H \equiv \sim P\sim$, respectively.

Finally, Prior established what he called the 'PF-calculus' by adding the following two axioms:

PF1: $p \supset GPp$

PF2: $p \supset HFp$

In his lecture, Prior demonstrated that the 'PF-calculus' is a rather powerful tool. As a nice example, he showed that the following is a theorem in the system:

$(p \vee Pp \vee Fp) \supset FPp$

Clearly, several other theorems can be proved in the system. In much of his later work with tense-logic, Prior concentrated on the exploration of what can be proved within the 'PF-calculus' and within other similar systems. However, in his presidential address in 1954, Prior emphasized that we may also discuss the logic of time in terms of another important formalism, namely, the so-called 'l-calculus', i.e., 'later than calculus' (1958a, 113). This alternative approach is based on the idea that time is a set of instants ordered by a before-after-relation, $(TIME, <)$.

For a modern reader, it may be surprising that Prior does not mention McTaggart's A- and B-series, which obviously correspond closely to the 'PF-calculus' and the 'l-calculus', respectively. However, as it appears from the preface of *Past, Present and Future,* Prior thought of McTaggart 'as an enemy' (1967, vi) until Peter Geach convinced him to revise his view.

In his presidential address in 1954, Prior pointed out that tenses may be introduced and further explored in terms of the 'l-calculus' using the following definition along with classical quantification theory:

$T(x,Fq) = \exists y\colon x<y \ \&\ T(y,q)$

$T(x,Pq) = \exists y\colon y<x \ \&\ T(y,q)$

This appears to suggest that tense-logic is just a by-product of the 'l-calculus'. However, in his lecture, Prior maintained that the metaphysics of time should in fact be conceived in the opposite manner:

> For 'now' is not the name of a date (it has the same meaning whenever it is used, but does not refer to the same date when it is used). In fact, the whole movement of events from the future through the present into the past is inexpressible in the l-calculus. If there is to be any 'interpretation' of our calculi in the metaphysical sense, it will probably need to be the other way round; that is, the l-calculus should be exhibited as a logical construction out of the PF-calculus rather than vice versa. (1958a, 116)

This view probably surprised the audience at the congress in Wellington, but the idea of the primacy of tense remained a cornerstone in Prior's philosophy of time until his death in 1969. As we shall see in Section 7, Prior introduced the so-called instant propositions to develop and support this idea.

In his lecture, Prior made it very clear that his formalism was designed to facilitate the exploration of some basic and classical problems within the metaphysics of time. In particular, the 'PF-calculus' should make it possible to study the problem of future contingency in a formal manner. Actually, the lecture marked the beginning of an extensive list of papers and book chapters dealing with the problems of (in)determinism, divine foreknowledge and human freedom in terms of Prior's 'PF-calculus' (i.e. his tense-logic).

At the end of the first day of the congress when Prior had given his important lecture on tense-logic, he wrote a letter to his wife explaining how it had been to present his important ideas at the congress:

> Darling, It's 2.15 a.m., & I'm at last in bed at the end of the 1st day of Congress, wh. has gone very pleasantly…… I put up my formulae on blackboard & started organising last night-&-this-morning's party; & then when the hour was due, delivered my piece. I felt very laboured in giving it, but was assured that it didn't look that way… (Prior 2022b)

Prior's main contribution to the study of time is his development of the 'PF-calculus' as a formalism from which the 'l-calculus' can be constructed. He had earlier, as a teenager, welcomed Bergson's intuitive ideas on time as a relevant response to determinism and the view of time as space. However, as a mature logician, he emphasized that much more is needed if we want to establish a proper and precise approach to the study of time and tense. In an undated note, he wrote:

> And I think it important that people who care for rigorism and formalism should not leave the basic flux and flow of things in the hands of existentialists and Bergsonians and others who love darkness rather than light, but we should enter this realm of life and time, not to destroy it, but to master it with our techniques. (Prior 2022c)

It should be noted that according to Prior, the concept of time as presented in terms of the 'PF-calculus' is very much like the understanding of time assumed in medieval logic (e.g. by William of Ockham), whereas the understanding of time as presented in terms of the '1-calculus' is very close to the idea used in medieval theology (e.g. by Thomas Aquinas). In his own words,

> Time, one might say, figures in the 1-calculus not as it does in medieval logic (which, as we have pointed out earlier, took tenses far more seriously than our own common logic does, and which already had such laws as our PF1), but rather as it does in medieval theology, in which God is said to behold all events in an unchanging present. (1958a, 117)

It appears that Prior's discussion of time according to medieval logic versus time according to medieval theology had given rise to some debate at the congress. In the letter to his wife, Prior wrote:

> There was a very pugnacious priest at the back who said that he was 'a Thomist & a strict Thomist', that this was the first exhibition he had seen of 'logistics', & that (this very aggressively & totally irrelevantly) he wanted to know if I was a 'realist'. I had a great deal of pleasure in telling him that I was far more of a realist than he was, & that he would in fact classify me as an 'extreme' realist. (2022b)

It should be mentioned that it is also evident from Prior's presidential address in 1954 that he knew his view on time may be seen as controversial, particularly by physicists and philosophers working with the key notions in Einstein's theories of relativity. It appears that Prior found that the burden of bothering with this discussion would be a necessary price to pay if we wanted to insist on proper freedom and indeterminism. At least there are interpretations of Einstein's special theory of relativity, which he wanted to question. He wrote:

> At least in many of its presentations, relativity theory seems to be as closely bound up with the 'spread-out-eternally' view of time underlying the l-calculus as medieval theology was. (1958a)

Prior clearly wanted to establish a formalism based on some fundamental philosophical assumptions on time and tense. In his undated note, *Some Free Thinking about Time*, he presented his basic beliefs in the following manner:

> …. what we see as a progress of events is a progress of events, a coming to pass of one thing after another, and not just a timeless tapestry with everything stuck there for good and all...
>
> This belief of mine... is bound up with a belief in real freedom. One of the big differences between the past and the future is that once something has become past, it is, as it were, out of our reach—once a thing has happened, nothing we can do can make it not to have happened. But the future is to some extent, even though it is only to a very small extent, something we can make for ourselves.... (Prior 2022d)

## 5. Further analysis of the Master Argument and similar arguments

As mentioned above Prior's study of the Master Argument of Diodorus played a very important role in his early development of tense-logic. Clearly, he found that the formal analysis of the Master Argument and similar arguments makes it possible to handle the struggle with the problems of determinism in a very precise and helpful manner.

Already in 1954, he wrote a paper suggesting a possible formalisation of the argument. This paper was published in (1955b). In (1958b) he published a new paper on the argument correcting a minor error in (1955b). In his very important book, *Past, Present and Future* (1967), Prior continued his work with the argument. Here he used a slightly different translation or paraphrase of the argument than the one found in (Mates 1953) claiming that the following three propositions cannot all be true (1967, 32):

D1. Every true proposition concerning the past is necessary.

D2. The impossible does not follow from the possible.

D3. Something that neither is nor will be is possible.

Obviously, the Master Argument was originally used as an argument in favour of determinism, i.e., given the validity of the trilemma, D3 must be rejected if D1 and D2 are accepted. This means that everything that neither is nor will be, turns out to be impossible. In other words, if something is the case and always will be, it is necessary (i.e., it could not have been otherwise).

If we let $L$ stand for 'it is necessary that …' and $M$ for 'it is possible that …' D1–2 becomes rather easy to represent in terms of Prior's tense-logical formalism:

D1. $Pq \supset {\sim}M{\sim}Pq$

D2. $L(p \supset q) \supset ({\sim}Mq \supset {\sim}Mp)$

If the argument is valid, it should be possible to demonstrate the denial of D3 based on the assumption of D1 and D2. The denial of D3 can be represented in the following manner:

D3'. $(\sim p \wedge \sim Fp) \supset \sim Mp$

However, Prior suggested that two additional assumptions are needed to establish a valid argument corresponding to Diodorus' ambition:

D4. $L(p \supset HFp)$

D5. $(\sim p \wedge \sim Fp) \supset P\sim Fp$

D4 means that if something is the case, it follows that it has always been that it would be going to be the case. D5 means that if something is false and always will be false, then it has already been the case that it would always be false. D4 and D5 may be assumed to be intuitively valid in a Diodorean context, although they are not explicitly mentioned as premises of the Master Argument. Furthermore, Prior was able to refer to recent historical research showing that D4 can be found 'in ancient' writers (1967, 33) and that D5 holds if time is discrete (1967, 49).

Prior proved D3' from D1, D2, D4 and D5 in the following way (Prior 1967, 33):

1. $(\sim p \wedge \sim Fp) \supset P\sim Fp$ (D5)
2. $P\sim Fp \supset \sim M\sim P\sim Fp$ (by D1 and substitution)
3. $(\sim p \wedge \sim Fp) \supset \sim M\sim P\sim Fp$ (by 1 and 2)
4. $L(p \supset \sim P\sim Fp) \supset (\sim M\sim P\sim Fp \supset \sim Mp)$ (by D2 and substitution)
5. $L(p \supset \sim P\sim Fp)$ (D4)
6. $\sim M\sim P\sim Fp \supset \sim Mp$ (by 4 and 5)
7. $(\sim p \wedge \sim Fp) \supset \sim Mp$ (by 3 and 6)

Consequently, at least one of the premises (D1, D2, D4 and D5) must be rejected to avoid the deterministic or even fatalistic conclusion, i.e., D3' (stated in 7).

In his study of the logical problems concerning determinism Prior also considered a similar argument formulated in terms of metrical tense operators $P(n)$, i.e., "it was the case $n$ time units ago that", and $F(n)$, i.e., "it is going to be the case in $n$ time units that" (Prior 1967, 119):

a. $P(m)p \supset LP(m)p$ (assumption)

b. $P(m)F(m+n)p \supset LP(m)F(m+n)p$ (by a and substitution)

c. $F(n)p \supset P(m)F(m+n)p$ (by e and f)

d. $F(n)p \supset LP(m)F(m+n)p$ (by b and c)

e. $L(p \supset q) \supset (Lp \supset Lq)$ (assumption)

f. $L(P(m)F(m+n)p \supset F(n)p)$ (assumption)

g. $LP(m)F(m+n)p \supset LF(n)p$ (by e and f)

h. $F(n)p \supset LF(n)p$ (by a and g)

There are obvious similarities between this argument and the Master Argument of Diodorus. The assumptions (a) and (c) are clearly very close the premisses, D1 and D4, respectively. Furthermore, the assumption (e) is basically the same premiss as D2. In his chapter on 'Time and determinism' (1967,113 f.) Prior also discussed other versions and aspects of the Master Argument. As we shall see, all this led him to the presentation of two possible responses to the attack on the doctrine of free choice to which the Diodorean argumentation may give rise.

## 6. Branching time

On 3 September 1958. Saul Kripke, who was then only 17 years old, wrote a letter to Prior. Kripke had read *Time and Modality*, 'with considerable interest' (Ploug and Øhrstrøm 2012). Among other things, Kripke wanted to comment on Prior's claims regarding the modal logic we obtain from a tense-logic if we take $Mp$ to stand for '$p$ is or will be the case'. In his book, Prior maintained that S4 is the modal logic that in this way comes out of his Diodorean tense-logic. Kripke demonstrated that this is wrong, and he pointed out that Prior's error has to do with an assumption regarding the notion of time assumed in his reasoning.

In his letter, Kripke argued that if we want a tense-logic corresponding to S4, a linear concept of time will be insufficient, and we should in fact base the analysis on a more complex notion of time. He wrote:

> Now, in an indetermined system, we perhaps should not regard time as a linear series, as you have done. Given the present moment, there are several possibilities for what the next moment may be like—and for each possible next moment, there are several possibilities for the next moment after that. Thus, the situation takes the form, not of a linear sequence, but of a 'tree'… (see [Ploug and Øhrstrøm 2012, 374])

According to Kripke, the temporal structure is backward linear and forward branching. In his letter, he illustrated this idea in the following manner:

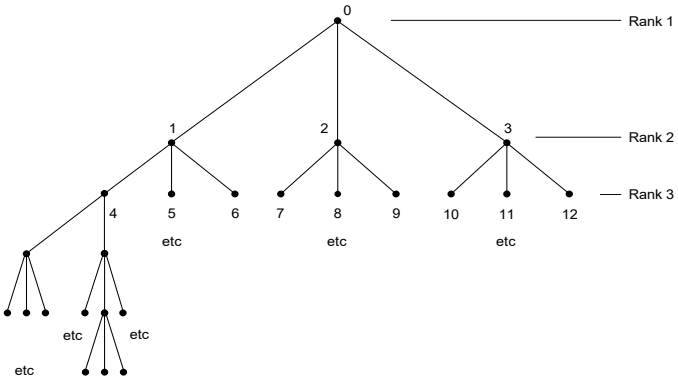


Fig. 1

Prior almost immediately accepted Kripke's idea of branching time. It is, in fact, likely that notions of this kind were well known to him. In fact, he might have known that Henri Bergson (1859–1941), in his book *Time and Free Will* (1910), had suggested a similar tree-like structure in his discussion of time and human decisions. It can even be argued that Prior already in 1957 had worked with a notion like branching time when he wrote his paper 'Opposite Number' (1957b), see (Øhrstrøm and González 2022). However, none of these earlier considerations included an account of the kind of branching time semantics suggested in Kripke's letter.

In the following years, Prior further developed the use of branching time models to give a precise account of the semantics of the tense-logical systems

he had in mind. In particular, this was important in cases in which he was unable to present the axioms of the systems.

Graphically, Prior turned Kripke's branching time diagram (Fig. 1) 90 degrees to have the future to the right and the past to the left.

One of the systems he found fascinating was the system inspired by the work of William of Ockham. He even added new elements to the branching time diagrams. First, he wanted to allow reference to time metrics according to a specific time unit (e.g. days) in the diagrams. In a draft, *Postulate Sets for Tense Logic*, written and circulated in 1965 or earlier, he also suggested a reference to 'a single designated line' in the diagram:

> In these models, the course of time (in a rather broad sense of this phrase) is represented by a line which, as it moves from left to right (past to future), continually divides into branches, so that from any given point on the diagram there is a unique route backwards (to the left; to the past) but a variety of routes forwards (to the right; to the future). In each model, there is a single designated point, representing the actual present moment; and in an Occamist model, there is a single designated line (taking one only of the possible forward routes at each fork), which might be picked out in red, representing the actual course of events. (Prior 2022f)[2]

Prior further developed this approach in a paper published the following year (Prior 1966). In this paper, Prior stated that in each Ockhamist[3] model, 'there is a single designated route from left to right, taking one direction only at each fork. This represents the actual course of events' (1966, 157). The idea is illustrated in Fig. 2, which $x$, $y$, $z$, $t$ are moments, and -$x$-$y$-$t$- and -$x$-$y$-$z$- are routes (sometimes called chronicles). As indicated -$x$-$y$-$t$- is the designated line (chronicle). This means that $z$ represents a possible moment at $y$ different from the chosen one. According

[2] It appears that that Prior had an idea very close to the notion of "the thin red line" that Belnap and Green (1994) independently (re)invented and criticized almost three decades later. - I owe this observation to Alex Malpass. See (Malpass 2011).

[3] It should be noted that Prior when preparing *Past, Present and Future* changed his spelling of the name of the famous medieval logician from 'Occam' to 'Ockham'.

to the system, propositions should primarily be evaluated relative to moments belonging to the designated line, and the other lines (routes, chronicles) are used to account for statements involving modal operators. According to the Ockhamistic logic a proposition, $p$, is necessary if and only if 'it is beyond our power to make $p$ false' (Prior 1966, 157), i.e., if and only if it is 'now-unpreventably' that it is true (Prior 1967, 117).

Given these ideas, Prior was able to present a formal account of the Ockhamist answer to the challenge of the Master Argument of Diodorus (conceived in the manner presented above). The Ockhamistic response consists in the rejection of the general validity of D1 (here understood as 'a' in the argument mentioned at the end of section 5). The point is that from an Ockhamistic point of view, D1 does not hold for statements formulated in the past tense about the future. To demonstrate what this means, we may consider diagram in Fig. 2, in which is obvious that $P(n)F(n+m)p$ is true at $y$, whereas $LP(n)F(n+m)p$ is false at $y$, since it was possible $n$ time units ago that $\sim p$ would be the case *n+m time units* later, namely at $z$. This clearly means that D1 does not hold at $y$. Consequently, it is also evident that the deterministic conclusion of the Master Argument can be avoided given an Ockhamistic system.

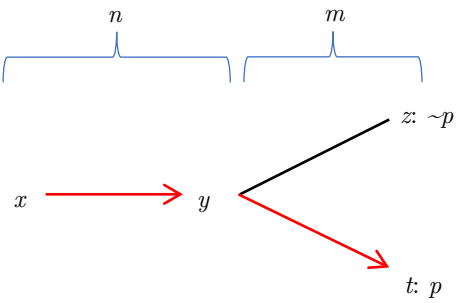


Fig. 2

For some reason, Prior decided to present a different formalisation of the Ockhamistic approach in his *Past, Present and Future* (1967). Here, there is no mention of a designated line corresponding to the actual course of events. Instead, a truth value is understood relative to a pair of a route in

the diagram and a moment belonging to the route. In fact, it turns out that we, in this way, will obtain the same theorems as according to the 1966 approach. However, the philosophical aspects of the notion of truth presented in the book clearly differ from the understanding of truth presented in 1965/66. In fact, we may speak of two different formalisations of Ockhamism: Ockhamism-1966 and Ockhamism-1967. The difference is that the former contains a reference to a designated line representing the actual course of event, whereas the latter does not contain any such reference. Historically, Ockhamism-1966 seems to be a much fairer representation of Ockham's original ideas than Ockhamism-1967. At least, it is obvious that William of Ockham held that God truly foreknows what is going to happen in the contingent future. A claim of this kind cannot even be made in terms of Ockhamism-1967.

D1 will turn out to be invalid, regardless of whether we accept Ockhamism-1966 or Ockhamism-1967. In Prior's opinion, the rejection of D1 was very problematic. He found that if something was true, we must accept this as a necessary (i.e., now-unpreventable) fact. As Prior wanted to hold on to D1 and as he wanted to avoid the fatalistic consequences of rejecting D3, he had to deny one of the premises D4–5. To solve the problem, Prior introduced a tense-logic, the so-called Peirce system, which differs from the Ockhamistic-1967 system in the use of a future operator that corresponds to the Ockhamistic *LF*. It is obvious from a diagram like Fig. 2 that this means that D4 must be rejected. It is also clear that the Peircean understanding of the future would leave no room for the idea of a designated route corresponding to the actual course of events.

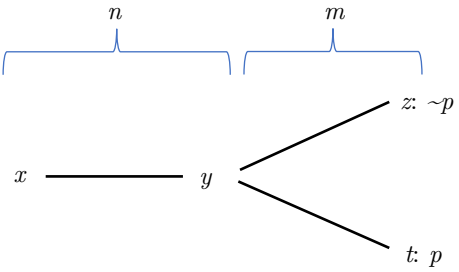


Fig. 3

From a Peircean point of view, a future tense proposition will only be true at a moment in a branching time diagram if it is true no matter which future branch we consider (Prior 1967, 128 ff.). Relative to Fig. 3 above, this means that $F(m)p$ and $F(m)\sim p$ are both false at $y$, whereas $\sim F(m)p$ and $\sim F(m)\sim p$ are both true at $y$. It has often been pointed out that it may appear counterintuitive to distinguish between $\sim F(m)\sim p$ and $F(m)p$.

Furthermore, it has been criticised that the Peirce system identifies 'it is going to be the case that …' and 'it is necessarily going to be the case that…'. Obviously, this approach seems to ignore important distinctions in natural language. On the other hand, it has been argued that if something is true about tomorrow, there must be something already now to make it true; therefore, what is going to be true must depend on some present truth. Still, it is not easy to precisely explain what a truth maker is. In fact, the discussion about truth makers can quickly become rather complicated; see e.g. (Craig 2001), (Merricks 2007) and (Tulenheimo 2020).

## 7. The understanding of the instants of branching time structures

Having worked with branching time structures for some years Prior wanted to give a precise account of the conceptual and ontological status of such structures and their components. How should the instants (moments) and chronicles (lines) in the branching time diagrams be understood? According to Prior, the instants and the chronicles in the diagrams should not be conceived of as objectively existing. They are nothing more than helpful constructions. In his *Past, Present and Future* (1967) and even more in his *Papers on Time and Tense* (1968), he explained how these constructions are carried out. In (1967, 79 ff. &187 ff.) he discussed the so-called world-state propositions, and in (1968, 122 ff.) Prior gave a very important account of his idea of seeing instants as a specific class of propositions. This work gave rise to an important new development of tense-logic. After Prior's death this work has been continued in the development of so-called hybrid logic, which has now grown into an important discipline that has become useful in computer science.

Prior's basic idea is that, in branching time logic, we should in fact operate with two different kinds of propositions (1968, 122 ff.). In addition to the usual tense-logical propositions formed based on atomic constants using various tense-logical operations, there is a special class of so-called instant propositions, $a$, $b$, $c$ …., with some extremely remarkable properties.

These very special properties of the instant propositions can be presented in terms of the following three axioms where $a$ is an arbitrary instant proposition and where $p$ is an arbitrary proposition in the logic:

(I1) $\exists a\colon a$

(I2) $\sim L \sim a$

(I3) $L(a \supset p) \vee L(a \supset \sim p)$

It is obvious from I1 that we must extend formal language with a quantification theory that allows propositional quantification over instant propositions.

The intuitive meaning of I1–3 is rather clear. I1 simply states that there is an instant proposition that is true (right now). Actually, we might call this instant proposition *Now*. I2 states that they are all possible instants and that may be conceived as past, future or even counterfactual. I3 means that for any instant proposition, $a$, and any tense-logical proposition, $p$, either $p$ or $\sim p$ follows necessarily from $a$. Intuitively, we may think of $L(a \supset p)$ as the claim that '$p$ is true at $a$'. If we substitute $a$ with *Now* in I3, the obvious reading becomes that any tense-logical proposition, $p$, will be either true or false at the present moment.

It is obvious that the use of instant propositions adds significantly to the expressibility of formal language. It is also clear that the instant propositions have some very remarkable properties. In fact, it turns out that everything in the whole branching time system will follow logically from the very rich information hidden in just one instant proposition. This means that in a certain sense, the Now includes everything that has been, will be, could be true, or could have been.

## 8. The thin red line

Since Prior's death in 1969, many tense-logicians have discussed the notion of the future within branching time semantics. One of the first philosophers to do so was Robert P. McArthur (1974). Later, others like J.R. Lucas (1989) made important contributions to the understanding of the so-called actual future. In a sense, this was a discussion very close to Prior's early studies on Ockhamism, in which he had considered the notion of a designated line corresponding to the actual course of events. However, in a very influential paper, Nuel Belnap and Michael Green (1994) criticised the idea of what they called 'the thin red line'.

Belnap and Green (1994, 379) pointed out that, in a branching time diagram, it will not work just to assume that there is a single designated line representing the actual course of events. If we can speak of a true future at some moments, it should be possible from all moments in the diagram. All moments should be treated in the same way. In consequence, if there are thin red lines from some of the moments in the diagram, there must be a thin red line from any moment in the diagram. Belnap and Green introduced a formal solution in the following manner:

> Technically, we change *TRL* from a simple name of a history to a function, $TRL(m)$, which picks out a unique Thin Red Line for each moment, $m$. (Belnap and Green 1994, 380)

It seems that Belnap and Green have a powerful case here. They have also argued that a notion like the suggested *TRL* function will be inconsistent with branching time semantics. However, as we shall see this part of their argument is rather problematic.

At an arbitrary moment, $m$, in the diagram $TRL(m)$ will be the line including the past, present and future relative to $m$. Belnap and Green (1994, 380) have pointed out that $m$ therefore will have to belong to $TRL(m)$, i.e.

(TRL1) $\quad m \in TRL(m)$

Furthermore, Belnap and Green argued that, if we want, some intuitively reasonable theorems such as $PPq \supset Pq$ and $FFq \supset Fq$ are valid; we must

make sure that *TRL* has the property of a certain kind of stability, which can be formulated in the following manner:

(TRL2) $m_1 < m_2 \supset TRL(m_1) = TRL(m_2)$

The idea is apparently that if $m_2$ is a moment later than $m_1$ then the future of $m_2$ must also be the future of $m_1$. However, if this requirement is accepted, it is easy to see that a diagram such as Fig. 3 will collapse into a linear structure. The reason is that the use of TRL2 on the combination of $y < z$ and $y < t$ will imply that $TRL(y) = TRL(z) = TRL(t)$, which means that $y$, $z$ and $t$ will all belong to the same line.

However, a defender of the idea of the thin red line cannot accept TRL2. Instead, we will have to do with the following weaker requirement:

(TRL2') $(m_1 < m_2 \wedge m_2 \in TRL(m_1)) \supset TRL(m_1) = TRL(m_2)$

This condition had in fact much earlier been suggested by Thomason and Gupta (1980). It turns out that TRL2' is enough to ensure the validity of $PPq \supset Pq$ and $FFq \supset Fq$. When this was communicated in 1996 to Nuel Belnap, the authors revised the claim they had made in (Belnap and Green 1994):

> I think you are quite right in bringing forth (2') in place of (2). This is not something that we had thought of and counts as a definite oversight on our part. Its consideration much improves the level of discussion (Personal e-mail from Nuel Belnap, 1 August 1996).

In their very influential book *Facing the Future* (2001), Belnap et al. took TRL2' into account, accepting that the TRL approach is logically possible, although they, for philosophical reasons, still found it problematic. One major formal criticism of the approach was that a semantics based on branching time diagrams with TRL functions does not include $q \supset HFq$ as a valid theorem. To verify that this is so, we consider the diagram shown in Fig. 4, assuming that $q$ is true at $j$ and nowhere else in the diagram. Since the assumptions mean that $Fq$ must be false at $i$, it obviously follows that $HFq$ is false at $j$ and that the same holds for $q \supset HFq$. Consequently, $q \supset HFq$ cannot be a valid theorem if the diagram is accepted semantically.

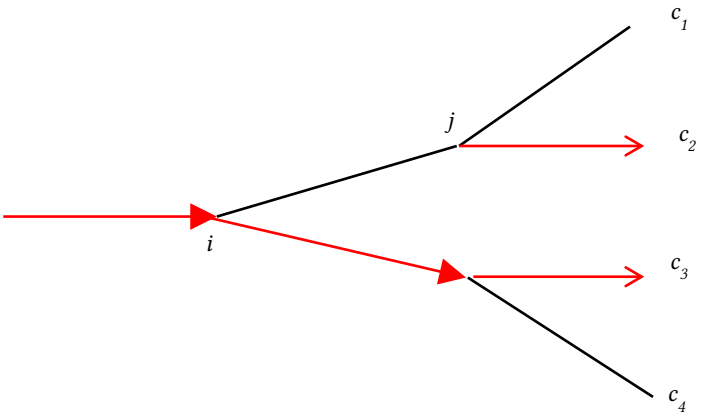


Fig. 4. The proposition q is supposed to be true at j, but nowhere else in the diagram.

It should be remembered that $q \supset HFq$ is a rather well-known and much discussed statement (see PF2 in section 4 above). As we have seen, it also played an important role in Prior's reconstruction of the Master Argument. Prior himself rejected it as invalid in general. So maybe we do not need it as a theorem. On the other hand, it is intuitively attractive, since most people will hold that if something is the case now, then it has always been going to be the case. For this reason, it may be reasonable to look for modifications to the TRL semantics according to which it is valid. One such attempt has recently been discussed by several authors (see [Øhrstrøm and Hasle 2020]). It is based on the definition of a revised TRL function for any arbitrary moment $j$ in the branching time diagram. This function is based on the general TRL function and is defined in the following manner:

$$i<j \supset TRL_j(i) = TRL(j)$$

otherwise $TRL_j(i) = TRL(i)$

This means that $TRL_j(i)$ only differs from $TRL(i)$ for moments $i$ that do not belong to the past of $j$. For moments belonging to the past of $j$, $TRL$ and $TRL_j$ will give us the same line (chronicle) in the diagram.

Furthermore, we introduce the notion of duration. Because the branching time system is backward linear, we can assume that there is a function, *back*, from TIME × N to TIME, where N is the set of natural numbers, such that *back*(*i*,*n*) is the unique instant *n* time units earlier than *i*.

Similarly, there is a function, *forward*, from TIME × TIME × N to TIME, such that *forward*(*i*,*j*,*n*) is the unique instant, *i'*, such that *back*(*i'*,*n*) = *i* and $i' \in TRL_j(i)$.

The idea is then to evaluate the tense-logical propositions relative to the moment of reference, *i*, as well as the moment we are giving priority in the actual context, *j*. Intuitively, we may think of *j* as the 'time of utterance'.

We define the truth value of a tense-logical proposition *p* at the instant *i* giving temporal priority to the instant *j*, *val*(*i*,*j*,*p*), recursively in the following way:

*val*(*i*,*j*,*p*)=1 iff *p* is a propositional letter assigned with the truth value 1 at the instant *i*, no matter to which moment we give temporal priority.

*val*(*i*,*j*,*P(n)p*)=1 iff *val*(*back*(*i*,*n*),*j*,*p*)=1.

*val*(*i*,*j*,*F(n)p*)=1 iff *val*(*forward*(*i*,*j*,*n*),*j*,*p*)=1.

(Negation and propositional connectives are treated in the usual manner.)

Furthermore, it is assumed that a tense-logical proposition is valid if and only if it is true at an arbitrary instant, *i'*, calculated by giving temporal priority to the same instant, *i'*. For instance, if we want to determine whether the proposition $q \supset P(n)F(n)q$ is true, we have to evaluate $val(i',i',q \supset P(n)F(n)q)$ at any moment *i'*. Let us again use *Fig. 4* as an illustration, assuming that *q* is true at *j* and that *i* is *n* time units before *j*. Clearly, $val(j,j,P(n)F(n)q) = 1$ if and only if $val(i,j,F(n)q) = 1$. To calculate the truth value of $F(n)q$ at *i* giving priority to *j*, the definition means that we have to use $TRL_j$ in the evaluation, which implies that *q* has to be evaluated along $TRL(j)$, i.e., $c_2$. It follows that $val(j,j,q) = 1$ if and only if $val(j,j,P(n)F(n)q) = 1$. Consequently, it is easy to see that $q \equiv P(n)F(n)q$, and using some basic quantification theory, it is easy to verify that $q \supset HFq$ is a theorem in the system.

It is interesting that the logic of the TRL approach, and in particular the just-mentioned version of it, appears to come very close to the logic of future contingency suggested by Luis de Molina (1535–1600), who wanted to show that the doctrines of divine foreknowledge and human freedom of choice do not contradict each other (see [Øhrstrøm and Hasle 2020]).

## 9. Conclusion

The modern study of tense-logic and its applications includes several aspects other than those mentioned above. However, the list of topics discussed in this paper probably suffices to demonstrate that tense-logic is a very rich field. During the seven decades since Prior's first studies in the area, several ideas and theories have been developed and even more interesting questions on time and modality have been asked. Many questions are still open—both regarding the formal properties of the systems and concerning the conceptual, philosophical and sometimes even metaphysical aspects of tense-logic. What makes Prior's tense-logic so great and remarkable is that his paradigm reaches far beyond his own findings during the 15 years he got to lay the foundation of the field.

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RESEARCH ARTICLE

# Fulfilling Russell's Wish: A.N. Prior and the Resurgence of Philosophical Theology

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*Abstract.* 'Wolterstorff (2009) provides an important explanation to the question: What caused the surprising resurgence of philosophical theology that has occurred over the last 50 years—a resurgence that rivals its zenith in the Middle Ages? This article supplements that with a more fine-grained answer to the question. Recent discoveries in Arthur Norman Prior's correspondence with J.J.C Smart and Mary Prior, between November 1953 and August 1954 on the possibility of necessary existence, demonstrates the importance of Prior's discussion of the Barcan formulae in *Time and Modality* (1957) for the resurgence of analytic theology. The correspondence establishes that Prior's discovery of tense-logic, and his discussion of quantified tense-logic constituted the perfect opportunity for him to challenge key anti-metaphysical assumptions in analytic philosophy, from which four important consequences can be drawn for the resurgence of philosophical theology. First, Prior's discussion of time and existence challenged the idea of Russell (1945) and Findlay (1948) on the logical status of a necessary existing being. Second, the discussion challenged the Analytic school's view of analysis and gave Prior the opportunity to introduce a different perspective on the relationship between logic and metaphysics. Third, it gave Prior a good

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opportunity to demonstrate that the then-prevailing attitude towards medieval logic was wrong. Fourth, it made it possible for Prior to demonstrate that the highly surprising metaphysical conclusions of quantified tense-logic brings modern logicians into a discussion with the theologically minded medieval logicians.

*Keywords*: A.N. Prior; analytic theology; the ontological argument; quantified tense-logic.

## 1. Russell's wish

In *History of Western Philosophy,* Bertrand Russel looked down from the pinnacle of his achievements in analytic philosophy. From this perspective he saw very little chance of discussing theology in the way it was done by medieval thinkers. This caused him to reflect on his preference for the old theology over the new.

> For my part, I prefer the ontological argument, the cosmological argument, and the rest of the old stock-in-trade, to the sentimental illogicality that has sprung from Rousseau. The old arguments at least were honest: if valid, they proved their point; if invalid, it was open to any critic to prove them so. But the new theology of the heart dispenses with argument; it cannot be refuted, because it does not profess to prove its points. At bottom, the only reason offered for its acceptance is that it allows us to indulge in pleasant dreams. This is an unworthy reason, and if I had to choose between Thomas Aquinas and Rousseau, I should unhesitatingly choose the Saint. (Russell 1945, 694)

Russell had, of course, played an important role in dispelling the medieval theologians from modern philosophy. His view on how to analyse philosophical problems was, by 1945, one of the leading principles in the analytic tradition now known as 'analytic philosophy'. According to him: "all philosophical problems—under a correct analysis—will be found to be either not philosophical or to be logical, 'in the sense in which we are using the word, logical' (Russell 1914, 33)." To Russell, the ontological argument was an example of how analysis can end philosophical discussion:

> Take, as a second example, the ontological argument. This, as we have seen, was invented by Anselm, rejected by Thomas Aquinas, accepted by Descartes, refuted by Kant, and reinstated by Hegel. I think it may be said quite decisively that, as a result of analysis of the concept 'existence', modern logic has proved this argument invalid. This is not a matter of temperament or of the social system; it is a purely technical matter. (Russell 1945, 786–87)

It is remarkable and an unexpected turn of events, considering Russell's words from 1945, that the last 40 years have seen a resurgence in philosophical theology in the analytic tradition, comparable only to that of the Middle Ages. Analytic theology, or Philosophical Theology as Wolterstorff (2009) call's it, is of relatively recent origin in the analytic tradition of philosophy. Most akin to systematic theology, it typically differs by placing the same kind of emphasis on analysis of concepts and propositions as that done by analytic philosophers. What caused this resurgence? According to Wolterstorff (2009) there are two main explanations: i) the downfall of logical positivism and, with it, the idea that there is a limit to the thinkable and assertible, and ii) the emergence of meta-epistemology. These coarse-grained explanations are hard to disagree with. They should however be supplemented with more fine-grained answers that demonstrates the connection between the understanding of 'analysis' as it is used in the field of analytic theology and as it is used by the founders of analytic philosophy. A first clue is perhaps already found in Russell's early awareness of Leibniz importance. One of the fundamental ideas within analytic philosophy is the view that 'all sound philosophy should begin with an analysis of propositions', which Russell considered 'a truth to evident, perhaps, to demand a proof' (Russell 1992, 9). He was, however, aware that this idea was not new but could be traced back at least to Leibniz. It raises the question whether analytic philosophy owes more to the past than Russell was prepared to accept. Commenting on Russell's acknowledgement of Leibniz, Michael Beaney asks: 'How far can we go back? To Descartes? To Ockham, Buridan, and other medieval logicians? To Aristotle or even Plato?'(Beaney 2013, 11) Perhaps Russell—and, with him, those who 'make logical analysis the main business of philosophy'(Russell 1945, 835)—had unwittingly signed up to a programme of philosophy that was more at home with Aquinas than

with the philosophical and theological tradition of the so-called Enlightenment?

This suggests a need to supplement Wolterstorff's (2009) explanation with a third point regarding how important it was for analytic theology that the attitude among logicians and philosophers towards medieval philosophy and logic proved to be wrong and based on ignorance. Recent research into the correspondence between Arthur Norman Prior and J.J.C Smart and Mary Prior, has led to new discoveries which demonstrates an important connection between his discussions of time and existence, in *Time and Modality* (1957), and necessary existence in *Is Necessary Existence Possible* (1955). It turns out, that Prior began writing the latter towards the end of 1953, just before the discovery of tense-logic, and that he struggled with anti-metaphysical assumptions in analytic philosophy, when he attempted to publish his article. These letters reveal that Prior struggled with problems relevant for the return of analytic theology and that he, in tense-logic, discovered a way to solve them which accomplished four important things for the resurgence of philosophical theology in analytic philosophy. First, it challenged Russell's assumption that modern logic had shown the ontological argument to be invalid as a mere technical matter concerning existence. Second, it challenged the Analytic school's anti-metaphysical view of analysis and introduced Arthur and Mary Prior's perspective of the relationship between logic and metaphysics. Thirdly, it demonstrated that the then-prevailing attitude towards medieval logic was wrong. Fourth, and finally, it demonstrated that highly surprising metaphysical, if not theological, conclusions are suggested by accepting the uncontroversial logical axioms of quantified tense-logic.

## 2. Prior's analysis of necessary existence

From Prior's correspondence with J.J.C. Smart, also known as 'Jack' Smart, and with his wife, Mary Prior, we see how, in the years leading up to the discovery of tense-logic, Prior struggled with the then-prevailing paradigm of what Skorupski (2013) calls the 'Analytic school', defined as 'a distinctive school of twentieth-century philosophy which focuses on the idea that the analysis of language is basic to philosophy as such: basic, moreover,

in a particular way—as the route by which traditional philosophical questions can be revealed as pseudo-problems' (Skorupski 2013, 299). Russell's rejection of the ontological argument, as a mere technical problem concerning existence, was an excellent example of what such a perspective could accomplish. This medieval argument had been debated for almost a thousand years and then it turns out to rest on a mere technical matter of logic! It would seem, to use Prior's words, that existence was "tied up and pub in a bag" (Prior 1976, 61). The paradigm of analysis of the analytic school was clearly visible in the policy statement of the journal *Analysis*, founded in 1933:

> The contributions to be published will be concerned, as a rule, with the elucidation or explanation of facts, or groups of facts the general nature of which is, by common consent, already known; rather than with attempts to establish new kinds of facts about the world, of very wide scope, or on very large scale. (Beaney 2013, 43)

As pointed out by Beaney, *Analysis* was 'one of the flagships of analytic philosophy' (Beaney 2013, 43). Logical analysis should, in principle, not yield any new kinds of fact about the world. This idea is clearly spelled out by Rudolf Carnap in *The Elimination of Metaphysics Through Logical Analysis of Language* (1932):

> The development of *modern logic* has made it possible to give a new and sharper answer to the question of the validity and justification of metaphysics. The researches of applied logic or the theory of knowledge, which aim at clarifying the cognitive content of scientific statements and thereby the meanings of the terms that occur in the statements, by means of logical analysis, lead to a positive and to a negative result. . . . In the domain of metaphysics, including all philosophy of value and normative theory, logical analysis yields the negative result that the alleged statements in this domain are entirely meaningless. Therewith a radical elimination of metaphysics is attained, which was not yet possible from the earlier anti-metaphysical standpoints. (Carnap 1959, 60)

A consequence of the then-prevailing view on the relationship between logic and metaphysics is evident in Findlay's argument against God's

existence in *Can God's Existence be Disproved?* (1948) John N. Findlay, under whom Prior's completed his Master of Arts (MA) in Philosophy in 1937, argued that because the idea of 'necessary existence' does not make sense and is an essential part of the concept of God, God cannot exist. Evidently, Findlay's argument rested upon the view of Russell, quoted above, that an analysis of existence could demonstrate that the ontological argument rests upon a mistaken view of existence, which renders it self-evidently absurd to talk about such a being:

> For if God is to satisfy religious claims and needs, he must be a being in every way inescapable, One whose existence and whose possession of certain excellences we cannot possibly conceive away. And modern views make it self-evidently absurd (if they don't make it ungrammatical) to speak of such a Being and attribute existence to him. (Findlay 1948, 182).

The dismissal of the ontological argument as resting on a technical mistake stood as a hallmark of what a correct analysis is capable of—namely, eliminating metaphysics. So much so, that an argument from the modern Russellian view on existence, according to Findlay could disprove the existence of God. It was, Findlay writes, "an ill day for Anselm when he hit upon his famous proof. For on that day he not only laid bare something that is of the essence of an adequate religious object, but also something that entails its necessary non-existence." (Findlay 1948, 182). Towards the end of 1953 however, Prior turned his attention to Russell's view on the concept of existence asking the question: is necessary existence possible? He had come to see a way in which G.E. Moore's and F.L.G. Frege's analysis of existence claims could be used to argue that the idea of necessary existence makes sense. His work led to one of his important analytic contributions to the field of philosophical theology, *Is Necessary Existence Possible* (1955). It occurred to Prior that in Frege's logic, propositions such as, 'Unicorns do not exist' must be rephrased as, 'The concept "unicorn" is not instantiated'. Preferring to talk of 'exemplification', Prior therefore noted that while some concepts do not preclude their own exemplification (such as 'unicornhood'), other concepts do (such as 'being at once cubical and non-cubical'). This means that the non-existence of unicorns differs from the non-existence of 'being at once cubical and non-cubical'; the first is a contingent fact, while

the other a necessity. His analysis leads him to raise the obvious question regarding God's supposedly necessary existence: why should there not be properties of concepts that necessitate their exemplification?

> We might then say that while it is a contingent fact that lions exist, since there is nothing about the concept of lionhood which necessitates its exemplification, it is a necessary fact that there is a God, since there *is* something about the concept of deity which necessitates its exemplification. (Prior 1955, 546)

It turns out, that Prior had a hard time getting his work on necessary existence published. We know from correspondence between Prior and Smart in 1953 that Prior had already sent an early version of it to the journal *Analysis* in December 1953. Unfortunately, in most cases, we only have the letters from Smart to Prior, but judging from Smart's letter to Prior on 15 November 1953, Prior had also clearly sent an early draft of *Is Necessary Existence Possible?* to him. It is clear from Smart's reply to Prior that the central discussion concerns the correct analysis of 'there exists a *y*' and that Smart's views on the matter cohered with that of the analytic school:

> Thank you for the necessary existence thing. Did I ever send you my lecture on the existence of 'God'? In this I argue that 'Logically necessary being' is self-contradictory like 'round square', simply because 'there exists a y' can never be a truth of logic. Your sentence 'For what cannot be thought of as attaching to a subject at all cannot be thought of as attaching necessarily to a subject' seems to me [to] miss the point. For clearly 'exists' <u>can</u> be predicated of God, unicorns, lions, etc. (Even though there is a sense in which 'it isn't a predicate'!). (Smart to Prior, 15 November 1953)

Unfortunately, we do not have Prior's response to Smart, but judging from his argument in *Is Necessary Existence Possible?*, Prior most likely offered reasons to reject the view that 'there exists a *y*' is not a truth of logic, based on Moore's and Frege's understanding of logic. Smart's reply came on 23 November 1953:

> Your defence of the ontological argument is immune to my criticism. I suppose deep down I just know there can't be any such property of concepts as you envisage because I have a conventionalist metaphysics engrained in me! But I must say it is difficult to find a knock down proof of the contradiction of your thesis. You ought to send it to some journal and see if the big brains can find a hole in your reasoning! (Smart to Prior, November 23, 1953)

It is interesting that Smart described Prior's paper as 'his defence of the ontological argument', as Prior, in his published version in 1955, writes that he does 'not wish to consider whether there is in fact any necessary being' (Prior 1955, 545). Prior was aware that his analysis of necessary existence would be seen as a defence of the ontological argument, but we do not have any reason to think that this is a characterisation he used himself in his letters to Smart. He did however on several occasions write about the ontological argument and considered the validity of modal as well as non-modal versions (Jakobsen & Øhrstrøm 2017). Mary Prior, who also had an MA in Philosophy and often discussed philosophy and logic with her husband in their correspondence, refers to it as, 'your necessary existence thing' in a letter to her husband while she was hospitalised with tuberculosis (Mary to Arthur Prior, August 17, 1954). Indeed, in one of the few letters we have from Prior to Smart, dated 30 June 1954, Prior writes about 'my defence of the possibility of Necessary existence'.

Prior took Smart's advice and sent his paper to the journal *Analysis*. On 7 December 1953, Smart writes that he is 'glad you are putting up the Nec. Connection thing to be shot at. It ought to create a lively discussion in Analysis'. From Smart's letter to Prior on 3 February 1954, we learn that Prior's article was rejected by *Analysis*. We also learn that Smart, in unmistakeable terms, disagreed with the arguments given for the decision. He found the decision taken by the editor narrow minded and marred by an "anti-metaphysical bias in the wrong sort of way." Despite his former words on the matter, Prior's "note on the ontological argument [sic] is a piece of analysis. And a much more interesting piece of analysis than the dull, and often quite mistaken, stuff so frequently published in Analysis." (Smart to Prior, February 3, 1954) As hinted by Smart, Prior could not

have failed to be aware that his defence of the notion of necessary existence would cause a lively discussion in *Analysis*, but neither he nor Smart had anticipated that the article would be rejected as not being an analysis. The discussion of what constitutes a logical analysis was often debated in the journal's first volumes following its founding in 1933 (Beaney 2013, 43). But something more than another theory on analysis had to happen. Indeed, the exchanges between Prior and Smart and Arthur and Mary Prior on the analysis of necessary existence show us what had to happen. Logical positivism was dying, but the hope obviously still lingered throughout the 1940s and early 1950s that logical analysis constitutes a demarcation between the medieval view on logic and philosophy and the modern view, perceived along the lines of the Analytic School. *Analysis* was a journal that saw itself as being in the business of guarding against the traditional view of logical analysis, as a tool to help draw out the implications of our metaphysical commitments about reality. We know that Arthur and Mary Prior discussed these paradigmatic matters with regard to Arthur's work on necessary existence. In fact, it turns out that Mary put words on the relationship she saw between Prior's 'necessary existing thing', which she considered 'a paradigm of philosophical argument' and what logical formulation can do:

> I've been thinking about your necessary existence thing and drawing morals from it. It seems to me to be a paradigm of philosophical argument. I mean the argument against has a philosophical rigour which objections like 'what Q. could that answer?' just haven't. It would be rash to claim that to any philosopher it is clear that the argument is no good because there are people who'll object to logic itself! But its [*sic*] clear that to most philosophers of whatever school and its [*sic*] good to see a philosopher dealing w. an argument as an argument, and not simply brushing it up in order to secure his own particular 'school' against another. To be interested in 'what is' instead of 'what ism,' wh. is the curse. And that is what logical formulation can do so well—get philosophy into a common language and clear from the language of the cliques. . . . So much 'philosophical' argument consists of changing the subject instead of arguing it out and I think Berkeley and Hume did try to argue out specific problems. (Mary to Arthur Prior, August 17, 1954)

The rejection of Arthur's argument as not being an analysis was, according to Mary, in line with dismissing the ontological argument by asking, 'What type of question would that answer?' Such a reply is dissatisfying since the arguments, that have traditionally been raised against Anselm's argument, of course grant that a meaningful analysis can be made of existence and necessity. It is evident that Mary and Arthur sought for a paradigm shift in philosophy, and they saw formal logic as having a key role to play in helping philosophers give a genuine treatment to arguments instead of merely securing their own 'ism'. Formal logic could 'get philosophy into a common language', 'clear from the language of the cliques'. There can be no doubt that Arthur shared Mary's view, as he writes the following in *On Some Proofs of the Existence of God*:

> We take it for granted nowadays that we have Existence properly tied up and put in a bag, but I don't know. I don't see that it doesn't make sense to say 'This exists', though its sense is no doubt a kind of tautology; and I don't see that it doesn't make sense to say 'This doesn't exist' though *its* sense is no doubt a kind of contradiction. It certainly makes sense, as Moore pointed out some years ago, to say 'This might not have existed', and for all I know there may be, as the theological tradition affirms, objects of which this last is true and objects of which it is false. (Prior 1976, 61)

Arthur's discovery of tense-logic proved to constitute the perfect framework for this discussion of modality and existence, in which it could be clear that we do not have existence 'tied up and put in a bag' as a purely technical matter. Equally important, it would challenge the perspective of the analytic school on the relationship between logic and metaphysics and provide a golden opportunity to introduce a new perspective in line with his and Mary's convictions.

## 3. Prior's discussion of existence and modality

The topic of Prior's John Locke Lectures, given in 1956, was tense-logic, but his mission was wider, as the first lines of *Time and Modality* (Prior

1957, vii) make clear: 'These lectures are the expression of a conviction that formal logic and general philosophy have more to bring to one another than is sometimes supposed'. Two philosophical issues were given central attention: the 'master argument' and 'existence and time'. The master argument was a philosophical problem, which, to Prior, had existential importance and was something he had pondered since 1931, when he first discovered the philosophy and theology of Jonathan Edwards and, for a brief period, became a keen disciple of him (Jakobsen et al, 1931). The second topic related to his overall aim of demonstrating that, contrary to the prevailing opinion in analytic philosophy, there were still philosophical problems related to the concept of existence. Both topics would be central to the ongoing discussion of tense-logic. The first issue, the 'master argument', constitutes the beginning of the modern discussion of divine foreknowledge and human freedom, culminating in *Formalities of Omniscience* (1962), which is arguably the earliest best example of the analytic theology to come. The second issue constitutes an important contribution to the understanding of what existential import, if any, is entailed by truths about non-existing objects (including future, past and non-actual objects); Prior's discussion of this issue serves to undermine Russell's idea that the ontological argument pivots on what is a mere technical, non-philosophical matter concerning existence.

### *3.1 Tense-logic and quantification*

Prior's discovery of tense-logic was in many ways connected to the influence of his teacher J.N. Findlay (see Jakobsen 2021). Most important was the influence of Findlay's *Time: A Treatment of Some Puzzles* (1941), which led Prior to see that tenses should be treated as a modality along the lines of 'it is necessarily the case that $p$' and 'it is possible that $p$'. In this manner, the future becomes 'it will be the case that $p$' (or in symbolism, $Fp$), the past becomes 'it was the case that $p$' (or in symbolism, $Pp$) and the present tense is simply $p$. Accordingly, we can say, that $Fp$ and $Pp$ take us to the future or past time, respectively, at which $p$ simply is true. From the weak operators $F$ and $P$, it is possible to define two strong operators $H \equiv \sim P\sim$ 'it has always been the case that $p$', and $\mathrm{G} \equiv \sim F\sim$ 'it will always be the case that $p$'. *Prior's* work on tense-logic lead to the formulation of a

minimal tense-logic, known as $K_t$, in which we have the following axiom schemes:

(A1) $p$, where $p$ is a tautology of the propositional calculus.

(A2) $G(p \supset q) \supset (Gp \supset Gq)$.

(A3) $H(p \supset q) \supset (Hp \supset Hq)$.

(A4) $PGp \supset p$.

(A5) $p \supset GPp$.

It also includes the rule of modus ponens:

(MP) If $\vdash p$ and $\vdash p \supset q$, then $\vdash q$.

Furthermore, it features the rules RG and RH for introducing tense operators:

(RG) If $\vdash p$, then $\vdash Gp$.

(RH) If $\vdash p$, then $\vdash Hp$.

For Prior's discussion of time and existence, it is important that we in $K_t$ are able to prove thesis T6:

(T6) $H(p \supset q) \supset (Pp \supset Pq)$.

The proof is simple from A3 using transposition:

| | |
|---|---|
| $H(p \supset q) \supset (Hp \supset Hq)$. | A3 |
| $H(\sim q \supset \sim p) \supset (H\sim q \supset H\sim p)$. | $p/\sim q$, $q/\sim p$ |
| $H(p \supset q) \supset (\sim H\sim p \supset \sim H\sim q)$. | Transposition |
| $H(p \supset q) \supset (Pp \supset Pq)$. | Df. H. |

With regard to quantification, this means that if there *will be* a person who flies to Mars then there *is* a person who will be flying to Mars, or formally:

$F\exists x{:}\phi(x)$ 'It will be that there is someone who is flying to Mars'.

entails

$\exists x{:}F\phi(x)$ 'There is someone who will fly to the moon'.

This surprising result from Ruth Barcan's formulae challenge the idea that existence is a technical matter. To prove it, we need, in addition to $K_t$, the following rules for the quantifiers:

($\forall$1) If $\vdash \phi(x) \supset \beta$, then $\vdash \forall x{:}\phi(x) \supset \beta$.

($\forall$2) If $\vdash \alpha \supset \phi(x)$, then $\vdash \alpha \supset \forall x{:}\phi(x)$, for $x$ not free in $\alpha$.

To these two rules correspond two rules for the existential quantifier:

($\exists$1) If $\vdash \phi(x) \supset \beta$, then $\vdash \exists x{:}\phi(x) \supset \beta$ , for $x$ not free in $\beta$.

($\exists$2) If $\vdash \alpha \supset \phi(x)$, then $\vdash \alpha \supset \exists x{:}\phi(x)$.

With these, Prior demonstrated that $F\exists x{:}\phi(x)$ entails $\exists x{:}F\phi(x)$ in the following manner:

(1) $G\phi(x) \supset G\phi(x)$

(2) $\forall \mathrm{x}{:}G\phi(x) \supset G\phi(x)$ (1 and $\forall$1)

(3) $H(\forall \mathrm{x}{:}G\phi(x) \supset G\phi(x))$ (2 and RH)

(4) $P\forall \mathrm{x}{:}G\phi(x) \supset PG\phi(x)$ (3, MP and T6)

(5) $P\forall \mathrm{x}{:}G\phi(x) \supset \phi(x)$ (4 and A4)

(6) $P\forall \mathrm{x}{:}G\phi(x) \supset \forall \mathrm{x}{:}\phi(x)$ (5 and $\forall$2)

(7) $G(P\forall \mathrm{x}{:}G\phi(x) \supset \forall \mathrm{x}{:}\phi(x))$ (6 and RG)

(8) $GP\forall \mathrm{x}{:}G\phi(x) \supset G\forall \mathrm{x}{:}\phi(x)$ (7, MP and A2)

(9) $\forall \mathrm{x}{:}G\phi(x) \supset G\forall \mathrm{x}{:}\phi(\mathrm{x})$ (8 and A5)

(10) $\sim G\forall \mathrm{x}{:}\phi(\mathrm{x}) \supset \sim\forall \mathrm{x}{:}G\phi(x)$ (9 and transposition)

(11) $F\exists \mathrm{x}{:}\phi(x) \supset \exists \mathrm{x}{:}F\phi(x)$ (10 and $F = \sim G\sim$)

The conclusion (11) is as surprising as the axioms and rules are uncontroversial. Ruth Barcan had already discovered the formulae in 1946, but they had not in general been applied to metaphysics. Tense-logic was however ideally suited to this as Williamson points out, it "is no surprise that the metaphysical implications of [the Barcan formulae] first became visible

through their analogous in temporal logic." (Williamson 2013, 66).[1] From what appears to be a natural thing we wish to say in tense-logic concerning some future object, it follows that our existential quantifier ranges over future as well as present objects. Taken at face value, then, if we are ontologically committed to whatever $x$ we quantify over then, as a mere technical matter, tense-logic gets us back into a sempiternal ontology with an uncanny similarity to the tenseless universe it set out to abandon. Prior therefore found it disturbing that 'the dubiety of the Barcan formula is . . . transmissible to the entire structure of the tense-logic we have so far erected' (Prior 1957, 27). It was evident to Prior that the Barcan formula constituted a strong challenge to accepting tense-logic because 'the only ground one can think of for assenting to it would be a conviction that whatever is going to exist at some future time exist[s] already' (Prior 1957, 29). Therefore, to him, there was a choice between finding a way to reject the conclusions of the Barcan formula—to not take tenses seriously—or to revise the original postulates for tense-logic to ensure a better fit between tense-logic and quantification, so that 'we may be in a better position to compare tense-logic and tenseless logic and to make our choice between them' (Prior 1957, 28). To some, such as L. Jonathan Cohen, the problems discussed by Prior concerning quantification and tense-logic were a reason to reject tense-logic:

> If we insist on having a 'tense-logic' we must assume that some form of discourse is sempiternal; and perhaps such an assumption would have seemed a commonplace to many theologically-minded ancient and medieval logicians. Or, if we reject any such assumption, we must also reject the idea of 'tense-logic' and fall back on the timeless truth—evaluations of ordinary logic. What we can be sure about is that 'it is not good logic' to try and have it both ways, as Professor Prior seems to do—to adopt a 'tense-logic' but to repudiate the sempiternity-assumptions. (Cohen 1958, 268)

When Prior, in *Past, Present and Future* (1967), took up the discussion again, now summarising a decade of research on the problem, it was evident

[1] I am grateful to an anonymous reviewer for reminding me of the work of Williamson on this issue.

that logicians and philosophers had not, in general, followed Cohen's idea of dismissing tense-logic as not good logic. On the contrary, as pointed out by Jack Copeland, Prior's work in Britain—including his John Locke Lectures and subsequent colloquiums and visits around the country—'helped to revitalize British logic' (Copeland 1996, 6); as was evident in Prior's discussion of time and existence in *Past, Present and Future* (1967), tense-logic had inspired many other logicians to work within the field. Cohen's comment is, however, interesting, as it points to an early awareness of the theological and metaphysical implications buried within the discussion begun by Prior on time and existence. Accepting tense-logic brings us to the edge of what a modern-minded logician or philosopher can accept and suggests that Prior's turn to the ancient and medieval view of logic invites philosophers to the discussion that are open to the metaphysics of the theology-minded medieval logicians.

## 4. The medieval turn

When Findlay in 1948 argued that God's existence could be proven to be impossible it rested upon the assumption that modern logic had proven the concept of necessary existence to be senseless. When Prior, in *On Some Proofs of the Existence of God* (1976), wrote that some philosophers had put forward what they claimed to be a disproof of the existence of God, he quite likely had his former teacher in mind. Russell's and Findlay's views of existence demonstrate in a clear way how the Analytic school barred the way for a 'theologically minded' medieval mindset, to use Cohen's term, in analytic philosophy. This accentuates the importance of Prior's discussion of time and existence in quantified tense-logic. His analysis of what he considered 'the untidiest and most obscure part of tense-logic' (Prior 1967, 172) demonstrated that we do not have existence 'tied up and put in a bag', as assumed by Russell and Findlay. Prior's correspondence with Smart and Mary Prior about necessary existence, along with his attempts to publish his work on the subject in 1954, reveal how Prior's subsequent discussion of time and existence in quantified tense-logic helped him challenge the assumptions within the Analytic school about what *analysis* means, which had prevented him from getting his article published in the journal

*Analysis.* Contrary to the analytic school, Prior considered logical analysis to be compatible with metaphysics. In *Past, Present and Future* (1967), Prior compares his understanding of the relationship between logic and metaphysics to that of a lawyer and a client. The job of the logician is like that of a lawyer, 'not in Toulmin's sense, that of reasoning less rigorously than a mathematician—but in the sense that he is there to give the metaphysician, perhaps even the physicist, the tense-logic that he wants, provided that it be consistent' (Prior 1967, 59). Prior became able to challenge the paradigm of analysis adhered to by the Analytic school because the discovery of tense-logic forces us to take metaphysics seriously in relation to the nature of time and logical realism. Furthermore, taking tenses seriously demonstrates that the logical analysis of time and existence, far from eliminating metaphysical problems, opens them up for metaphysical and further logical analysis. When this, as pointed out by Cohen, brings us back to the theologically minded medieval thinkers, it is because tense-logic is fundamentally a strong defence of the medieval and ancient view of the tensed nature of propositions. Willard Van Orman Quine had argued in 1953 that one has not really appreciated what modern logic is if one does not see that it must be tenseless (Quine 1953). Against Quine, Prior had argued the following:

> There are no grounds of a purely logical character for the current preference, and . . . 'propositions' in the ancient and medieval sense lend themselves as readily to the application of contemporary logical techniques and procedures as do 'propositions' in the modern sense. (Prior 1958, 105)

The turn to medieval logic not only challenged Quine's view of what formal logic is but also served a greater purpose for Prior, who wanted to change the prevailing attitude within analytic philosophy towards ancient and medieval logic:

> Neither Russell nor, I think, [Alfred North] Whitehead brought to their work on mathematical logic any very close or detailed acquaintance with the logic of Aristotle and the Schoolmen. Toward Aristotelianism and scholasticism [Russell's *sic*] attitude has always been one of contempt; and his example has helped to make it customary in English-speaking countries for modern

> mathematical logic and the Aristotelian logical tradition to be set in sharp contrast to one another. (Prior 1951, 46)

Prior's discovery of tense-logic therefore constitutes an important part of answering Wolterstorff's question. It challenged the view on medieval logic and thinking regarding such a fundamental questions as, 'What is a proposition?'. To the medieval logician, as Uckelman points out, all logic was temporal logic (Uckelman 2013, 485), but as Øhrstrøm and Hasle point out, this assumption lost its influence in the humanistic critique of scholastic logic (Øhrstrøm and Hasle 1995, 85). Although historians of medieval philosophy appreciated analytic method prior to Prior (see e.g. Salamucha (1934/1969), or Boehner (1952))[2], the demonstration that formal logic is not necessarily tenseless was a discovery of great importance for the return of medieval philosophy and brought about a significant shift to analytic philosophy and the history of medieval philosophy.

The resurgence of medieval theology was already apparent to Prior. It was evident to him that his analysis of time and existence suggested—what Cohen had also pointed out—that perhaps a turn to the medieval view of logic implied a turn to the theologically focussed mindset of medieval logicians concerning facts about non-existent objects. Medieval logicians had no problem accounting for the truth-conditions of propositions involving references to merely future or past objects. Prior was aware of this and included it in his analysis of time and existence when he returned to it in *Past, Present and Future* (1967):

> The idea of a permanent pool of objects, some now existing and some only having existed or going to exist, seems to be presupposed in the medieval theory of ampliatio, according to which what things a general term can stand for depends in part on the tense or mood of the verb with which it is used. In 'Some man is running', the word 'man' can stand for any man now existing; but in 'Some man will be running' it can stand also for a man who merely will exist, and in 'Some man could be running' it can stand for a man who merely could exist—in the one case,

[2] I am grateful to an anonymous reviewer of this article for pointing out the importance of Salamucha 1934/1969 and Boehner 1952.

> supponit pro futuris, and in the other pro possibilibus, and not only pro praesentibus. The metaphysics involved in this way of talking is apt to strike the modern reader as weird. . . . But let us not exaggerate this queerness. What this comprehensive objecthood amounts to is simply that there are already facts about these objects, even if they are not yet existent. (Prior 1967, 30–31)

Prior did not accept the medieval idea, and as such, it was not as a defender of this medieval perspective that he contributed to the resurgence of philosophical theology. He paved the road, however, for subsequent discussions of this topic by demonstrating that from simple axioms, quantified tense-logic yields conclusions that are metaphysically controversial to the modern mind but were generally accepted as valid by theologically minded medieval thinkers. Medieval logicians would, as Ernest Addison Moody (1953) points out, deny the idea so important to many modern philosophers that 'This term stands for something, therefore it stands for something which exist[s]' (Moody 1953, 57). Subsequent discussions of this problem of grounding for propositions about future, past, non-existing objects still seem to favour modern intuitions. There are however strong defenders of the medieval perspective described by Moody, such as Craig (2017).

## 5. Conclusion

Little known to Russell, less than a decade after his musings on Rousseau and Aquinas, Prior demonstrated, through his discussion of quantified tense-logic, that, contrary to the then-prevailing opinion in analytic philosophy, existence is not 'tied up and put in a bag'. His discovery of tense-logic proved Quine wrong on the idea that formal logic must be tensed and made it possible to tease out the metaphysical aspects of quantified tense-logic. The new discovery in this story comes from Prior's correspondence with Smart and Mary on the possibility of necessary existence. Here we see Prior's struggles with getting his work on the possibility of necessary existence published and the importance this question had for him and Mary. His discovery of tense-logic and his presentation of this in Oxford in 1956 at the

John Locke lectures provided the perfect opportunity for the discussion of time and existence in *Time and Modality* (1957) which would significantly promote his and Mary's paradigm for the relationship between logic and philosophy. It challenged, head on, the view of the journal *Analysis*—that a correct logical analysis should eliminate metaphysics and not suggest metaphysical conclusions from logical inquiry. The surprising results of quantified tense-logic has medieval logic written all over it, from the fundamental acceptance of the medieval and ancient view of propositions to the sempiternal conclusions already considered by medieval logicians in their theory of ampliatio. It paved the way for inviting theologically minded logicians of the Middle Ages into modern discussions of existence and future contingency. Prior, for these reasons, helped fulfil the wish of Russell to discuss theology with Aquinas rather than Rousseau. Additional work needs to be done to provide a more detailed, fine-grained answers to the important question: why did analytic theology appear in a tradition that had written it off as an absurdity?

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RESEARCH ARTICLE

# Impossible Events and the Knowability Paradox

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*Abstract*: This note disambiguates the predicate 'is an unknowable event' and shows how Transparent Intensional Logic interprets the sentences "Agent *a* is calculating the final decimal of π" and "Agent *a* has calculated the final decimal of π". The knowability paradox is used to set the stage.

*Keywords*: Impossibility; event; knowability paradox.

Are impossible events unknowable? We must distinguish between *impossible knowledge* and *knowledge of impossibility*. To explain the difference, we begin with a fact. It is an arithmetic fact that the decimal expansion of π does not terminate in a final number. Hence, nobody could have possibly calculated the final number of this series. Having calculated a number is understood to be tantamount to the successful completion of a computational process. It is an impossible event that somebody should do so. This is distinct from the possible event of somebody being in the (albeit non-

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terminating, hence inherently frustrating) process of calculating this final number. Being in this computational process does not presuppose the existence of a number with a particular property, such as being the final number of an infinite expansion. It does presuppose the existence of an (non-effective) algorithm that the agent is intentionally related to. This is analogous to being in the process of squaring the circle without there being squares equal in area to circles.

Nobody could possibly know that the impossible event of having successfully calculated the final number ever occurred, whereas somebody might know that somebody was in the process of calculating the final number. In the former case, it is not that there would be a kind of event that nobody could know about, but rather that there is simply nothing to know. Somebody can have knowledge of impossibility by knowing that it is (arithmetically) impossible to successfully complete the calculation. Somebody can also have knowledge of impossibility by knowing that nobody could possibly know that the event of somebody successfully completing the calculation had occurred.

An appeal to the factivity of knowledge suffices to make the point about there being nothing to know, with the added restriction that the sort of thing that is required as complement cannot possibly exist. This objective impossibility entails another objective impossibility, namely that there is no destination for the itinerary of a computational process to terminate at. Contrast this with subjective incapacity in the form of a restriction of a particular cognitive faculty: there is something 'out there' alright, only it is beyond epistemic reach on ground of principle.

We have just described two impossible events; one being predicated on the other:

- the final decimal of $\pi$ having been successfully calculated
- somebody knowing about the final decimal of $\pi$ having been successfully calculated

Is either of them an event, except one that could not possibly be realized at any possible world? Or are impossible events not events at all, but of a different nature, say, concepts or presentations of events? We claim that an impossible event is a particular kind of concept that could not possibly have

an instance. So, in this sense unrealizable events are unknowable, as they can have no instances, and so none could be known to be true. But in another sense, they are perfectly knowable. Once you know about an impossible event, you know at least some of what there is to know about a particular conceptualization of a particular kind of event. This approach is strictly top down and *ante rem*. It is not so that, in terms of conceptual priority, we start out with events and then work our way back to conceptualizations of them. We embrace conceptualizations of events, such that these conceptualizations could not possibly have an instance.

The theory of impossible events being presupposed is a counterproposal to the standard modal Meinongian take on *impossibilia* such as impossible events. We do not frontload impossibilities, which would include impossible events that nonetheless occur somewhere in logical space. We do not require that one must try to make sense of a *number* that would be the final one in the expansion of π. Rather we are, so to speak, elevating the impossible objects of Meinongianism to concepts while jettisoning the category of objects instantiating such concepts. Our position is a concept-first account of impossibilities and the epistemic access to them. Impossible events should not be misconstrued as impossible *realia*, as events unfolding at impossible worlds. Talking of impossible events as events that have the property of being impossible is akin to talking of fake banknotes as banknotes that have the property of being fake. The problem with this is that no set of banknotes includes any that is a fake banknote: being a fake banknote is not a property that a banknote can instantiate.[1] Analogously, events are typed to take place within some empirical dimensions (for their part, formalized as modal and temporal parameters), so it is inherent to an event to be alethically (or 'metaphysically') possible. An impossible event (whatever it is) scores a zero in point of empirical realizability. But it is not nothing. It is an intentional (or 'ideal') object, in that it can be contemplated in thought and referred to in language.

The overall plan is this. We start out with a standard case bearing on unknowability, describe why it is not problematic for us, and use the case to ponder the nature of impossible events. We show why it is enlightening

[1] See Carrara et al. (2017) on privative modifiers.

to study a typed solution to the so-called *knowability paradox*. It is an inference whose conclusion is itself not a contradiction; rather it is an inference that takes ostensibly not-too-controversial starting points to a conclusion that is inconsistent with one of the assumptions. The upshot of the paradox is this: if every true proposition is knowable then every true proposition is known. Or by contraposition, if not every true proposition is known then some true proposition is unknowable (in standard notation):

$$\frac{\Diamond Kp \rightarrow Kp}{\neg Kp \rightarrow \neg \Diamond Kp}$$

Can it be blocked by an inherently (i.e., not *ad hoc*) typed epistemic logic? Yes, it can. Should each and every of the rules required for the paradox to succeed be accepted? No, not if we construe knowledge hyperintensionally.[2] The overall lesson is that a deduction such as the one underlying the knowability paradox is one that a theory of impossibility should not allow to get off the ground in the first place. The lesson is not that a theory should engage with the derivation, and then present ways to render the derivation invalid. It only gets off the ground, because it presupposes too crude a notion of objects of knowledge and of impossibilities. Or so our diagnosis goes. We engage with the knowability paradox, because it challenges us to reflect upon the nature of impossibility, including impossible events, and the potential for having knowledge about impossibility.

These are the building-blocks of the knowability paradox in its standard rendition: they are expressed in first-order propositional logic and its modal extension.

| | |
|---|---|
| Distribution | $K\,(p \wedge q) \vdash Kp \wedge Kq$ |
| Factivity | $Kp \vdash p$ |
| Necessitation | If $\vdash p$ then $\Box p$ |
| Interdefinability | $\neg \Diamond p \vdash \Box \neg p$, $\Box \neg p \vdash \neg \Diamond p$ |

[2] To construe knowledge hyperintensionally means to construe knowledge as a relation to a hyperpropositions, which in turn is a proposition that is individuated more finely than up to co-intensionality/necessary equivalence.

Knowability (UK) $\forall q\ (q \rightarrow \Diamond Kq)$

Ignorance (Ign.) $\exists r\ (r \wedge \neg Kr)$

UK is universal *knowability*, or the principle of knowability: every truth is knowable; no truth is such that it inherently eludes being known; every truth that obtains is possibly known by somebody somewhere, i.e., known at some index in logical space. If knowability is restricted to a subset of logical space, or even just one world, then UK seems overly optimistic. If knowability extends to all of logical space, then UK borders on triviality, as logical space must exhaust the logically possible. *Ignorance* is non-omniscience: at some index or other, some truth or other eludes being known by any member of the totality of epistemic agents. Variables $p$, $q$, $r$ range over propositions, which are just sets of worlds (or of world/time pairs). Accordingly, $K$ takes sets of worlds (or world/time pairs) as arguments. The first argument of $K$, the epistemic agent, is suppressed, as the agent is just an inert point of evaluation.

This is how the *knowability paradox* is generated. [4] is an instantiation of the possibility occurring at [3].

| | |
|---|---|
| [1] $p \wedge \neg Kp$ | instantiation of Ign. |
| [2] $p \wedge \neg Kp \rightarrow \Diamond K(p \wedge \neg Kp)$ | UK, 1 |
| [3] $\Diamond K(p \wedge \neg Kp)$ | MPP, 1, 2 |
| [4] $K(p \wedge \neg Kp)$ | *assumption* |
| [5] $Kp \wedge K\neg Kp$ | Dist., 4 |
| [6] $Kp \wedge \neg Kp$ | Fact., 5 |
| [7] $\neg K(p \wedge \neg Kp)$ | 4, 5, 6 |
| [8] $\Box\neg K(p \wedge \neg Kp)$ | *Nec.*, 7 |
| [9] $\neg\Diamond K(p \wedge \neg Kp)$ | *Interdef.*, 8 |

[9] is inconsistent with [3]. It is unacceptable that a set of principles and rules of inference should be able to generate an inconsistency. If the derivation is valid, there is something wrong with this set. One way to block the deduction would be to drop one of the two principles, either *ignorance* or

*universal knowability*.[3] The result is, respectively, that all truths are known (sooner or later), or that some truths are unknowable. A second way is to drop either distributivity or factivity. Dropping factivity is not an option, of course, as factivity is a formal feature of knowledge. However, sophistication is called for when including both hyperpropositions and truth-conditions in the same theory. Distribution is part and parcel of epistemic logic when erected on normal modal logic (though not in neighborhood semantics, for instance), but not obviously valid in hyperintensional epistemic logic. Of course, distribution is instrumental in generating the contradiction at [6] within the sub-proof that begins at [4] and ends at [7].

A third way, which we will be exploring here, does two things. First, it is developed within a hyperintensional framework that comes with a typed universe. The typing of levels, something which is objected to in Carrara and Fassio (2011), is not a superimposed addition *ad hoc*, but is inherent to the framework. Second, distribution does not hold in hyperpropositional attitude contexts, unless it is foisted upon them; but then the point of going hyperintensional would be undercut.

[3] An objection to the knowability paradox is based exactly on the idea that the interpretation of not least the principle of universal knowability is incorrect. Properly interpreted, the premises would not generate a contradiction, as the derivation of the argument would be blocked. Edgington (1985) proposes the first and best-known solution to the paradox using a semantic revision of this principle. She departs from a parallelism between a *temporal* and a *modal* formalization of the paradox. The basic idea is that that there could be agents that can know propositions with the form $p \wedge \neg Kp$. A possible knower in a non-actual situation could have counterfactual knowledge that ($p$ and nobody knows that $p$) is true in the actual situation. Consider the fact that the last non-avian dinosaur died in the year $Y$ and nobody knows that. An agent in a different possible world could discover that the last non-avian dinosaur died in $Y$, and could have counterfactual knowledge of a situation identical to the actual one in which nobody comes to know this fact. According to Edgington, this would amount to counterfactual knowledge of an actual truth of the form $p \wedge \neg Kp$. Along parallel lines, one may suggest that a subject could come to know truths like $p \wedge \neg Kp$ at a different time. Nobody knows now when the very last non-avian dinosaur died, but in the future someone could come to know that, and also know that nobody knew it at the time that happens to be our present time.

Carrara and Fassio (*ibid.*, 191) runs this argument against type-based stratification. [4*] has been correctly obtained, and its type levels check out, but [5*] has not been correctly obtained, as it does not follow from [4*], although distribution has ostensibly been applied correctly:

[4*] $K_2(p_0 \wedge \neg K_1 p_0)$

[5*] $K_2 p_0 \wedge K_2 \neg K_1 p_0$

Applying factivity to the second conjunct in [5*] yields [6*]:

[6*] $K_2 p_0 \wedge \neg K_1 p_0$

[6*] is not inconsistent, thanks to $K_2$ versus $K_1$. The problem with [5*], though, is that it violates the rule that the level of $K$ must be exactly one level up from the level of its operand, here $p_0$. Thus, when applying distribution to [4*], the correct result would instead have to be this:

[5**] $K_1 p_0 \wedge K_2 \neg K_1 p_0$

Applying factivity to [5**] yields

[6**] $K_1 p_0 \wedge \neg K_1 p_0$

which *is* inconsistent. So, if all typing amounts to is stratification, and distribution forces $K_2$ in [4*] to downgrade to $K_1$ in [5**] and [6**], then this just reveals that the framework is shallow. In particular, the difference in type is not indicative of any difference in granularity between the complements of $K_2$ and of $K_1$. This sort of typing does no more than track degrees of syntactic embedding.

By contrast, a type theory worth the name uses its types to indicate levels in granularity. These differences in granularity will, in turn, affect which derivations go through and which do not. The 'paradox' does not get started, once its 'derivation' has been transferred into Transparent Intensional Logic, where the relation of knowledge is construed as a binary relation-in-intension between an epistemic agent (the knower) and a hyperproposition. Here is a few rewrites to illustrate the point. Derivations require that hyperpropositions undergo λ-elimination (because valid derivations operate on truth-values, in order to preserve truth rather than

meaning), but we will display the pre-elimination forms to demonstrate the full forms of knowledge attributions.

Both the formal framework of Transparent Intensional Logic and its philosophical tenets will be presupposed.[4] The types involved are the following. K: *being known*, an empirical property of hyperpropositions/$(o^*_n)_{\tau\omega}$; $c/^*_2 \rightarrow {}^*_1$: $c$ is a second-level variable presenting a first-level hyperproposition (both of them higher-order objects); ${}^2c \rightarrow o_{\tau\omega}$: the hyperproposition presented by c presents a proposition (empirical truth-condition); ${}^2c_{wt} \rightarrow o$: the extensionalization of the so-presented proposition in order to obtain a truth-value. As is seen, four levels are involved, which are those of second-level higher-order object, first-level higher-order object, intensional first-order object, extensional first-order object. These levels do not vary with context, as the infelicitous typed 'solution' to the paradox does. Especially, the type of the argument of $K$ does not co-vary with embedding, but remains fixed.[5]

[1TIL] $\lambda w \lambda t\, [{}^0\wedge\, {}^2c_{wt}\, {}^0\neg[{}^0K_{wt}\, c]]$

This captures ignorance of a truth. The thing to note here is that the hyperproposition presented by variable $c$ occurs *displayed* as a hyperproposition in its own right rather than in *executed* mode, in which mode the hyperproposition serves to yield its product, a proposition.

[4TIL] $\lambda w \lambda t\, [{}^0K_{wt}\, {}^0[{}^0\wedge\, {}^2c_{wt}\, {}^0\neg\, [{}^0K_{wt}\, c]]]$

The thing to note here is that the Composition $[{}^0\wedge\, {}^2c_{wt}\, {}^0\neg{}^0K_{wt}\, c]$ occurs Trivialized, i.e. displayed. What is known is that the Composition produces a truth. What is not known, on pain of making a category mistake, is its product, which is a truth-value (namely, the truth-value that $\wedge$ yields as its functional value). Due to the Composition occurring displayed, every procedure occurring inside it also occurs displayed. Hence, the Double Execution ${}^2c_{wt}$ and the Composition $[{}^0\neg\, {}^0K_{wt}\, c\,]$ occur displayed as well. As a result,

[4] See, e.g., Duží et al. (2010), Jespersen and Duží (2022), Duží et al. (2023).

[5] Stating the factivity constraint is also a bit technically involved, because the type theory does not allow this easy inference: $Kp \vdash p$ ("what is known is true"). See Duží et al. (2010, §5.1.6).

they are 'frozen' and cannot be operated on directly within this embedding.[6] Hence, distribution does not kick in, as distribution is defined only for intensional contexts.

[5TIL] $\lambda w \lambda t\, [{}^0\wedge\ {}^0K_{wt}\, c\, [{}^0K_{wt}\ {}^0\neg\ [{}^0K_{wt}\, c]]]$

The thing to note about distribution is that it inverts the scope of $K$ and $\wedge$. Loosely speaking, distribution takes a compound attitude (knowledge of a conjunction) and turns it into two single attitudes conjoined by conjunction. Distribution is not valid in the epistemic logic of TIL, unless it is added as a stipulation that the (here, implicit) epistemic agent has sufficient logical intelligence to extract the two conjuncts occurring within the scope of $K$ and re-embed them individually in the scope of $K$ and, furthermore, always does so. Assuming [4] for negation introduction thus makes little strategic sense. All in all, TIL does not arrive at the conclusion $\neg\Diamond K(p \wedge \neg Kp)$, because when the derivation is translated into TIL, it comes out invalid. Therefore, TIL does not get to face the choice between *ignorance* and *universal knowability*, as these principles are defined by modal epistemic logic.

With the knowability paradox out of the way, in the sense that it cannot be generated and so does not affect the answer to the initial question as to whether impossible, or non-actualizable events, are knowable, we now turn to answering this question. TIL is a hyperintensional theory for the logic of the language by means of which we express ourselves. It is not a theory of the metaphysics of reality, say, of grounding or of events. An event is simply of the same type as propositions: $o_{\tau\omega}$. Therefore, there is just one impossible event, the one that never obtains anywhere in logical space. So, the action is elsewhere, namely, in the fine-grained, different conceptualizations of this one limiting-case intension.[7] Let us revisit the two cases we contrasted at the outset.

*Contingent truth* (CT) Agent $a$ is calculating the final decimal of $\pi$.

*Necessary falsehood* (NF) Agent $a$ has calculated the final decimal of $\pi$.

[6] See Jespersen and Duží (2022) on how to operate on displayed procedures.

[7] See Duží et al. (2021), which is the first TIL study devoted entirely to impossibility.

CT is an inherently futile endeavour, one that cannot meet with success, but it is no less of an endeavour for it. Its canonical form in TIL is this Closure:

(CT TIL) $\lambda w \lambda t\, [{}^0Calc_{wt}\, a\, {}^0[{}^0Final\, {}^0\pi]]$

*Final*/$(\nu\tau)$: a function taking a (transcendental) number to its last decimal digit; $\pi/\tau$. That is, the agent is related to a calculation of a natural number. The Composition $[{}^0Final\, {}^0\pi]$ is a procedure that does not terminate in a product, though the type theory specifies the type of the product which the procedure is structured and typed to produce, namely, $\nu$, i.e., the type of natural numbers. Schematically speaking, where the Meinongian would invoke an impossible number (the final number of the expansion of $\pi$), TIL invokes an 'impossible' procedure, one necessarily lacking a product. The Trivialization of this Composition, ${}^0[{}^0Final\, {}^0\pi]$, is the complement of *a*'s computational attitude: *a* is intentionally related to a procedure structured and typed to produce an object of type $\nu$. (CT) presupposes, without specifying any, that *a* is following an algorithm during the process of calculating the final decimal of $\pi$. *a*'s predicament is that while *a* understands the algorithm in question well enough for the computational process to take place, *a* has (not yet) figured out that the algorithm will not terminate in a number.

For a variation, consider this ascription of an attitude *de re*:

(CT*) The last decimal of $\pi$ is being calculated by *a*

An argument consisting in inferring from (CT*) the following conclusion is valid, but also necessarily unsound, because the premise makes the impossible presupposition that the last decimal of $\pi$ should exist:

(CT**) There is a number such that *a* is calculating it as the last decimal of $\pi$

(CT*) yields a truth-value gap: there is no such number around to make it true or else false that *a* (or whoever else) is in the process of calculating it. The conclusion is a necessary falsehood. Therefore, (CT*) describes an impossible event. However, the validity of the argument is impervious to mathematical facts; the argument has the right logical form to be valid. This inference has the following form in TIL, where the functions *Sub* and

*Tr* make the Composition [$^0$*Final* $^0\pi$] occur extensionally, as required by an attitude *de re*:[8]

(CT* TIL) $\lambda w \lambda t\, [{}^0Calc_{wt}\, a\, [{}^0Sub\, [{}^0Tr\, [{}^0Final\ {}^0\pi]]\ {}^0y\, [[{}^0Final\ {}^0\pi] = y]]]$

(CT** TIL) $\lambda w \lambda t\, [{}^0\exists \lambda x\, [{}^0Calc_{wt}\, a\, [{}^0Sub\, [{}^0Tr\, x]\ {}^0y\, [[{}^0Final\ {}^0\pi] = y]]]]$

$x/*_n \rightarrow_v \tau$; $Sub/(*_n *_n *_n *_n)$: substitution trades procedures for procedures within procedures, thus forming new procedures; $Tr/(*_n \tau)$: a function taking a number to its Trivialization.

The logical form of (NF) includes empirical indices (worlds, times), because *Calc* is a binary relation-*in-intension* between a calculating agent and a piece of mathematics. Consider this inference:

a has calculated the final decimal of π

---

a has calculated something

Again, the argument is valid, for sure, but also unsound, because the premise is (necessarily) false.

An important feature of NF is that it is expressed by means of the present perfect.[9] The point of evaluation (say, 1 April 2023) must be included in the interval of times, during which it is already a fact that *a* has completed their calculation. The sentence "*a* has calculated the final decimal of π" is not specific enough for a temporally sensitive analysis. The proper analysandum is instead "*a* has already calculated the final decimal of π in 2023". Its canonical form is this:

(NF.TIL) λwλt [0PfPrt [0Already$_{w}$ λw'λt' [0Has_Calc$_{w't'}$ a 0[0Final 0π]]] 02023]

*Types*: *PfPr*/((o(o(oτ))(oτ))τ); *Already*/((o(oτ))o$_{\omega\tau}$)$_\omega$; *2023*/(oτ); *Has_Calc*/(oι$*_n$)$_{\omega\tau}$ is a relation-in-intension between an individual and a procedure, such that the individual has successfully executed the procedure.

[8] See Duží and Jespersen (2017, §5.1). The analysandum contains just the phrase 'the *final* decimal of π', so this is all that gets carried through to the analysis. However, it is always an option to introduce a refinement specifying a particular manner, in which the function Final has been produced. For refinement, see Duží et al. (2010, 524-26).

[9] The present perfect is explained in Duží et al. (2010, §2.5.2.2).

The Closure (NF.TIL) produces the following *truth-condition*. The point of reference must include the present time of evaluation. The relation between a set of intervals $S/(\mathrm{o}(\mathrm{o}\tau))$ and an interval $I/(\mathrm{o}\tau)$ at a reference time $T/\tau$ is that $I$ must be an interval which runs from the past up to, and perhaps beyond, $T$, and $I$ is an element of $S$. This truth-condition cannot possibly be fulfilled, however. (NF.TIL) produces 'bottom', i.e., the unique proposition that returns the truth-value 0 at every world $w$ and every time $t$. This is due to the fact that in order for the truth-condition to be satisfied, the interval of $2023/(\mathrm{o}\tau)$ must be an element of the set $S/(\mathrm{o}(\mathrm{o}\tau))$ of intervals, in which the truth-condition produced by the Closure $\lambda w'\lambda t'$ $[{}^0\mathit{Has\_Calc}_{w't'}\ a\ {}^0[{}^0\mathit{Final}\ {}^0\pi]]$ is satisfied in the world $w$ and at the time $t$ of evaluation. Yet, $S$ is the empty set of intervals.

Finally, we return to some epistemic variations on CT and NF:

(ECT) Agent $b$ knows that $a$ is calculating the final decimal of $\pi$.

(ECT.TIL) $\lambda w\lambda t\ [{}^0\mathit{Know}_{wt}\ b\ {}^0[\lambda w\lambda t\ [{}^0\mathit{Calc}_{wt}\ a\ {}^0[{}^0\mathit{Final}\ {}^0\pi]]]]$

ECT is itself a contingent truth: $b$ happens to know that $a$ happens to be in the process of calculating the final decimal of $\pi$.

(ENF) Agent $b$ knows that $a$ has already calculated the final decimal of $\pi$ in 2023.

(ENF.TIL) $\lambda w\lambda t\ [{}^0\mathit{Know}_{wt}\ b\ {}^0[\lambda w\lambda t\ [{}^0\mathit{PfPr}_t\ [{}^0\mathit{Already}_w\ \lambda_w'\lambda_t'\ [{}^0\mathit{Has\_Calc}_{wt}\ a\ {}^0[{}^0\mathit{Final}\ {}^0\pi]]]\ {}^0\mathit{2023}]]]$

ENF is an instance of *impossible knowledge*, in the sense that there is no such thing as knowing such-and-such, because there could not possibly be any such-and-such.

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RESEARCH ARTICLE

# Specification of Agents' Activities in Past, Present and Future

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*Abstract*: The behaviour of a multi-agent system is driven by messaging. Usually, there is no central dispatcher and each autonomous agent, though resource-bounded, can make less or more rational decisions to meet its own and collective goals. To this end, however, agents must communicate with their fellow agents and account for the signals from their environment. Moreover, in the dynamic, permanently changing world, agents' behaviour, i.e. their activities, must also be dynamic. By communicating with other fellow agents and with their environment, agents should be able to learn new concepts and enrich their knowledge base. Processes and events that happened in the past may be irrelevant in the present or have a significant impact in the future, and vice versa. Therefore, the fine-grained analysis of agents' activities as well as events within or beyond the system is very important so that the system can run smoothly without falling into inconsistencies. Moreover, as the system should communicate with its environment, the analysis should be as close to natural language as possible. The goal of this paper is a proposal for such an analysis. To this end, I apply Transparent Intensional Logic (TIL) because TIL is particularly apt for a fine-grained analysis of processes and events specified in the present, past

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or future tense with reference to the time when they happened, happen or will happen.

*Keywords*: Activity; Communication of agents; Transparent Intensional Logic; Natural language processing; Sentences in different tenses.

## 1. Introduction

A multi-agent system (MAS) is a distributed system of (more or less) intelligent agents who are *active* in their perceiving environment and acting to achieve their individual and collective goals.[1] The agents are autonomous in the sense of not being controlled by a central dispatcher; the system is driven only by messaging.[2] To obtain a needed piece of information, the agents must be able to *ask* their fellow agents. Yet, they need to put forward not only *Yes-No* questions but also, in particular, *Wh*-questions. While there is just one type of answer to a Yes-no question, the class of Wh-questions is much more abundant in types. From the logical point of view, the type of possible answer determines the type of Wh-question. In regular communication, we ask by using different pronouns in interrogative sentences, and these pronouns indicate the type of possible answer. We can integrate logical and linguistic views to classify Wh-questions into more detailed classes. For instance, by 'who', we ask for a person; by 'where', for a location or position; 'when' means asking for the time. A proposal for such a more detailed classification of Wh-questions has been introduced in (Číhalová, Duží 2022). Each specialised subtype of a Wh-question conveys specific instructions for an agent on how and where to find the corresponding answer. Detailed classification of queries thus improves agents' communication and intelligent behaviour. In particular, the specific types of Wh-questions are apt for the communication of agents concerning their *dynamic*

[1] By 'intelligent' I do not mean human intelligence in case of software agents, of course. Instead, I am talking about artificial intelligence, which is actually not an intelligence, as Roger Penrose in his 1994 book argues. Anyway, in this paper I use the term 'intelligence' for both.

[2] See, for instance Wooldrige (2009).

*activities*. The agents need to know *who* is the actor of an activity, *when* the activity starts and ends, by *which* instruments it is performed, etc.

The systems of erotetic logic are valuable, as they render many exciting features of Yes-No questions and answers.[3] However, many other essential features of questions stem from their *presuppositions*. Yet, to my best knowledge, none of the systems of erotetic logic deals with Wh-questions and presuppositions of questions in a plausible way. This is unsatisfactory, as Wh-questions are even more frequent than Yes-No questions in our everyday vernacular.[4]

To obtain a literal analysis of natural language sentences, I am going to apply Tichý's (1988) Transparent Intensional Logic (TIL) with its procedural semantics, namely, its version as introduced in (Duží, Jespersen and Materna 2010). The analysis of empirical Wh-questions transforms in the TIL formalism into λ-terms denoting procedures that produce α-intensions (functions with the domain of possible worlds ω and times τ, and values of type α) where α is not a truth-value. The sought answer should provide an object of type α, which is the value of the α-intension asked for in the actual world at the time of evaluation. Since ordinary erotetic logics do not usually deal with Wh-questions, (Duží and Fait 2021) adjusted Gentzen's system of natural deduction for TIL so that the system can answer not only Yes-No questions by keyword searching but also answer Wh-questions by inferring computable knowledge from natural-language texts.[5] The paper

[3] See, for instance, Harrah (2002) or Peliš and Majer (2011). For a system based on relevant logic that can provide axioms and rules for dealing with Yes-No questions, see, for instance (Punčochář 2020).

[4] There are a few systems dealing with Wh-questions, see, for instance, Groenendijk (2003), Haida (2008), Hamblin (1973), Essberger (online) or Kartunen (1977). Yet, none of them covers this issue in a satisfactory way. Their summary and appraisal from the point of view of application in TIL can be found in Číhalová, Duží (2022).

[5] Computable or inferable knowledge has been introduced as a golden middle way between two extremes, namely explicit and implicit knowledge. Classical epistemic systems deal with explicit and implicit knowledge. The former prevents the paradox of logical/mathematical omniscience by depriving the agents of any inferential abilities, as they know only those pieces of knowledge that are explicitly recorded in their knowledge base. On the other hand, dealing with implicit knowledge

describes a useful logical technique of deriving answers to Wh-questions based on a given knowledge base that can be both an agent's base or even natural language texts. It consists of enriching the system of natural deduction with special rules rooted in the rich semantics of a natural language. In addition, special technical rules are specified to operate *into* hyperintensional contexts; see Duží, Jespersen (2015) and Jespersen, Duží (2022).

In (Číhalová, Duží 2022) the analysis of agents' activities is briefly outlined. The goal of this paper is to propose a detailed analysis of agents' dynamic activities both from the point of view of their specifications and answering questions on such activities. The analysis takes account of time, i.e. sentences in the past, present or future tenses with reference to the time when this or that happened, is happening or will happen.

The rest of the paper is organised as follows. Section 2 summarises the basic principles of Transparent Intensional Logic (TIL). In Section 3, I briefly reproduce the conceptual-oriented classification of *Wh*-questions as of (Číhalová, Duží 2022). The main novelty of this paper is presented in Section 4; it is the analysis of agents' dynamic activities specified in past, present or future tenses together with the agents' learning new concepts by questioning and answering. Concluding remarks and proposals for further research can be found in Section 5.

## 2. Basic Principles of TIL

Pavel Tichý, the Transparent Intensional Logic (TIL) founder, was inspired by Frege's semantic triangle. Frege characterised the sense of an

presupposes that the agents would be able to derive all the logical consequences of their explicitly recorded pieces of knowledge, if only they had an infinite amount of time and resources at their disposal. Hence, implicit knowledge inevitably yields the paradox of logical/mathematical omniscience. Since both notions are not realistic in case of modelling behaviour of intelligent but resource bounded agents, we introduce the notion of inferable knowledge. The idea is simple. Having an agent with some inferential abilities and an explicit knowledge base, we compute maximal limit of knowledge they are able to infer by applying the rules of inference the agent masters. For details, see Duží, Menšík (2017).

expression as the 'mode of presentation'. Tichý defines this mode of presentation as an abstract, algorithmically structured *procedure* that produces the object denoted by the expression or, in rigorously defined cases, fails to produce a denotation if there is none.[6] This is because there are non-denoting terms that have a perfect meaning, like 'the greatest prime number' or 'the value of the cotangent function at the number π'. Mathematicians had obviously to understand the sense of these terms first, and only then could they prove that there are no such numbers. Hence, in TIL, the meaning of an expression is understood as a context-invariant *procedure* encoded by a given expression. By 'context invariant', we mean this. The procedure encoded by an unambiguous expression is one and the same (up to procedural isomorphism) independently of the context in which the expression is used.[7] If the expression is ambiguous, it is furnished with more than one procedure corresponding to its different meanings.

Tichý defined six kinds of meaning procedures and called them *constructions.* There are two kinds of *atomic* constructions that supply input objects to be operated on by molecular constructions. They are *Trivialization* and *Variable.* A Trivialisation presents an object $X$ without the mediation of any other procedures. Using the terminology of programming languages, the Trivialisation of $X$, denoted by '$^0X$', is just a *pointer* or *reference* to $X$. Trivialization can present an object of any type, even another construction $C$. Hence, if $C$ is a construction, $^0C$ is said to *present* the construction $C$, whereby $C$ occurs *hyperintensionally*, i.e. in the *non-executed* mode. Variables produce objects dependently on valuations; they are said to *v-construct.* The execution of a Trivialisation or a variable never fails to produce an object. However, since TIL is a logic of partial functions, the execution of some of the molecular constructions can fail to present an object of the type

[6] See Tichý (1988). A similar philosophy of meaning as a 'generalized algorithm' can be found in (Moschovakis 2006); this conception has been further developed by Loukanova (2009). TIL procedural viewpoint is also not far from the idea of algorithmic logic, see Li, B. (2022). 4936. https://doi.org/10.20935/AL4936.

[7] For the definition of procedural isomorphism, see (Duží 2019). Briefly, there is no unique criterion for procedural isomorphism and any language, any discourse. In practice, procedures are isomorphic if their specification is identical up to α-equivalence or restricted β-equivalence.

they are typed to produce. When this happens, we say that a given construction is *v-improper*.

There are two kinds of *molecular* constructions, which correspond to *λ-abstraction* and *application* in the λ-calculi, namely *Closure* and *Composition*. *λ-Closure*, $[\lambda x_1 \ldots x_n\ X]$, is the very procedure of producing a function with the values $v$-produced by the procedure $X$, by abstracting over the values of the variables $x_1$, …, $x_n$ to provide functional arguments. No Closure is $v$-improper for any valuation $v$, as a Closure always $v$-constructs a function (which may be, in an extreme case, a degenerate function undefined at all its arguments). *Composition*, $[X\ X_1 \ldots X_n]$, is the very procedure of applying a function $f$ produced by $X$ (if any) to the tuple argument $\langle a_1, \ldots, a_n \rangle$ (if any) produced by the procedures $X_1$, …, $X_n$. A Composition is $v$-improper as soon as $f$ is a *partial function* not defined at its tuple argument or if one or more of its constituents $X$, $X_1$, …, $X_n$ are $v$-improper.[8]

TIL being a *hyperintensional* system, each construction $C$ can occur not only in execution mode so as to produce an object (if any) when being executed but also as an object in its own right on which other (higher-order) constructions operate. The Trivialisation of $C$ causes $C$ to occur just presented as an argument, as mentioned above. Yet sometimes, we need to cancel the effect of Trivialisation and trade the mode of $C$ for execution mode. *Double Execution*, ${}^2C$, does just that; it executes $C$ twice over. If $C$ $v$-constructs a construction $D$ that in turn $v$-constructs an entity $E$, then ${}^2C$ $v$-constructs $E$. Otherwise, ${}^2C$ is $v$-improper. Hence, for any construction $C$, this law is valid: ${}^{20}C{=}C$.

**Definition 1** (*construction*)

(i) *Variables* $x$, $y$, … are *constructions* that construct objects (i.e., elements of their respective ranges) dependently on a valuation function $v$; they $v$-construct.

[8] In the rest of this section, I draw on the standard exposition of the fundamentals of TIL, as presented in other papers (for instance in Jespersen, Duží (2022) or Duží, Fait (2021)), with just a few minor adjustments. True, since TIL has become a well-known system, this exposition could have been more condensed; yet, in the effort of making everything comprehensive and convenient for a reader, I leave this part in full details.

(ii) Where $X$ is an object whatsoever (even a *construction*), ${}^0X$ is the *construction Trivialisation* that constructs $X$ without any change.

(iii) Let $X$, $Y_1$, …, $Y_n$ be arbitrary *constructions*. Then the *Composition* $[X\ Y_1 \ldots Y_n]$ is the following *construction*. For any $v$, the Composition $[X\ Y_1 \ldots Y_n]$ is *v-improper* if one or more of $X$, $Y_1$, …, $Y_n$ are $v$-improper, or if $X$ does not $v$-construct a function that is defined at the $n$-tuple of objects $v$-constructed by $Y_1$, …, $Y_n$. If $X$ does $v$-construct a $v$-proper function, then $[X\ Y_1 \ldots Y_n]$ $v$-constructs the value of this function at the $n$-tuple.

(iv) (λ-) *Closure* $[\lambda x_1 \ldots x_m\ Y]$ is the following *construction*. Let $x_1$, $x_2$, …, $x_m$ be pair-wise distinct variables and $Y$ a *construction*. Then $[\lambda x_1 \ldots x_m\ Y]$ $v$-constructs the function $f$ that takes any members $B_1$, …, $B_m$ of the respective ranges of the variables $x_1$, …, $x_m$ into the object (if any) that is $v(B_1/x_1,\ldots,B_m/x_m)$-constructed by $Y$, where $v(B_1/x_1,\ldots,B_m/x_m)$ is like $v$ except for assigning $B_1$ to $x_1$, …, $B_m$ to $x_m$.

(v) Where $X$ is an object whatsoever, ${}^1X$ is the *construction Single Execution* that $v$-constructs what $X$ $v$-constructs. Thus, if $X$ is a $v$-improper construction or not a construction as all, ${}^1X$ is $v$-improper.

(vi) Where $X$ is an object whatsoever, ${}^2X$ is the *construction Double Execution*. If $X$ is not itself a *construction*, or if $X$ does not $v$-construct a *construction*, or if $X$ $v$-constructs a $v$-improper *construction*, then ${}^2X$ is $v$-improper. Otherwise ${}^2X$ $v$-constructs what is $v$-constructed by the *construction* $v$-constructed by $X$.

(vii) Nothing is a *construction*, unless it so follows from (i) through (vi).

With constructions of constructions, constructions of functions, functions, and functional values in TIL stratified ontology, we need to keep track of the traffic between multiple logical strata. The *ramified type hierarchy* discharges that task. The type of first-order objects includes all objects that are not constructions. Therefore, it includes not only the standard objects of individuals and truth values but also sets, functional mappings and functions defined on possible worlds (i.e., the *intensions* germane to possible-world semantics, PWS intensions). The type of second-order objects includes constructions of first-order objects and functions with such

constructions in their domain or range. The type of third-order objects includes constructions of first- or second-order objects and functions with such constructions in their domain or range; and so on ad infinitum.

**Definition 2** (*ramified hierarchy of types*)**.** Let $B$ be a *base*, where a base is a collection of pair-wise disjoint, non-empty sets. Then:

$\mathbf{T}_1$ (*types of order 1*).

i) Every member of $B$ is an elementary *type of order 1 over B.*

ii) Let $\alpha, \beta_1, ..., \beta_m$ $(m > 0)$ be types of order 1 over $B$. Then the collection $(\alpha\ \beta_1 ...\ \beta_m)$ of all $m$-ary partial mappings from $\beta_1 \times ... \times \beta_m$ into $\alpha$ is a functional *type of* order 1 *over B.*

iii) Nothing is a *type of order 1 over B* unless it so follows from (i) and (ii).

$\mathbf{C}_n$ (*constructions of order n*)

i) Let $x$ be a variable ranging over a type of order $n$. Then $x$ is a *construction of order n over B.*

ii) Let $X$ be a member of a type of order $n$. Then ${}^0X$, ${}^1X$, ${}^2X$ are *constructions of order n over B.*

iii) Let $X$, $X_1$, ..., $X_m$ $(m > 0)$ be constructions of order $n$ over $B$. Then $[X\ X_1 ...\ X_m]$ is a *construction of order n over B.*

iv) Let $x_1$, ..., $x_m$, $X$ $(m > 0)$ be constructions of order $n$ over $B$. Then $[\lambda x_1 ... x_m\ X]$ is a *construction of order n over B.*

v) Nothing is a *construction of order n over B* unless it so follows from $\mathbf{C}_n$ (i)-(iv).

$\mathbf{T}_{n+1}$ (*types of order n + 1*)

Let ${}^*_n$ be the collection of all constructions of order $n$ over $B$. Then

i) ${}^*_n$ and every type of order $n$ are *types of order n + 1.*

ii) If $m > 0$ and $\alpha, \beta_1, ..., \beta_m$ are types of order $n + 1$ over $B$, then $(\alpha, \beta_1, ..., \beta_m)$ (see $\mathrm{T}_1$ ii)) is a *type of order n + 1 over B.*

iii) Nothing is a *type of order n + 1 over B* unless it so follows from (i) and (ii).

For the purposes of natural-language analysis, we are usually assuming the following base of ground types:

- ο: the set of truth-values {**T, F**};
- ι: the set of individuals (the universe of discourse);
- τ: the set of real numbers (doubling as times);
- ω: the set of logically possible worlds (the logical space).

We assume that the universe of discourse ι is multi-valued and consists of at least two elements, though here I leave aside the cardinality of this basic type.

Empirical expressions denote *empirical conditions*, which may or may not be satisfied at the world/time pair selected as points of evaluation. These empirical conditions are modelled as (PWS-)intensions. *Intensions* are entities of type (βω): mappings from possible worlds to an arbitrary type β. The type β is frequently the type of the *chronology* of α-objects, i.e., a mapping of type (ατ). Thus α-intensions are mostly functions of type ((ατ)ω), abbreviated as '$\alpha_{\tau\omega}$'.[9] *Extensional entities* are entities of a type α where α ≠ (βω) for any type β. Where the variable *w* ranges over β and *t* over τ, the following outline of a Closure essentially characterises the logical syntax of empirical language: $\lambda w \lambda t$ [...*w*....*t*...].

*Examples* of frequently used α-intensions are: *propositions* of type $o_{\tau\omega}$, *properties of individuals* of type $(o\iota)_{\tau\omega}$, *binary relations-in-intension between individuals* of type $(o\iota\iota)_{\tau\omega}$, *offices* of type $\iota_{\tau\omega}$ and hyperintensional *attitudes* of type$(o\iota{*}_n)_{\tau\omega}$. Logical objects like *truth functions* and *quantifiers* are extensional: ∧, ∨, ⊃ are of type (ooo), and ¬ of type (oo).

[9] We define (PWS-)intensions as functions with the domain of possible worlds. True, most frequently, time plays the role of the second modal parameter, though not always. For instance, assuming that physical laws of nature are nomically but not analytically necessary, as physics is an empirical science, we model these intensions by construction of this form: $\lambda w\, \forall t$ [...] $\rightarrow o_\omega$.

The *quantifiers* $\forall^\alpha$, $\exists^\alpha$ are type-theoretically polymorphic total functions of type $(o(o\alpha))$, for an arbitrary type $\alpha$, defined as follows. The *universal quantifier* $(\forall^\alpha)$ is a function that associates a class $A$ of $\alpha$-elements with **T** if $A$ contains all elements of the type $\alpha$, otherwise with **F**. The *existential quantifier* $(\exists^\alpha)$ is a function that associates a class $A$ of $\alpha$-elements with **T** if $A$ is a non-empty class, otherwise with **F**.

*Notational conventions.* Below all type indications will be provided outside the formulae in order not to clutter the notation. Moreover, the outermost brackets of Closures will be omitted whenever no confusion can arise. Furthermore, '$X/\alpha$' means that an object $X$ is (a member) of type $\alpha$. '$X \to \alpha$' means that $X$ is typed to *v*-construct an object (if any) of type $\alpha$. Throughout, it holds that the variables $w \to \omega$ and $t \to \tau$. If $C \to \alpha_{\tau\omega}$ then the frequently used Composition $[[C\ w]\ t]$, which is the *extensionalization* of the $\alpha$-intension *v*-constructed by $C$, is encoded as '$C_{wt}$'. When no confusion arises, I am going to use the standard infix notation without Trivialisation for the application of logical objects like truth functions and quantifiers. Hence, instead of '$[^0\forall\lambda x\ B]$', '$[^0\exists\lambda x\ B]$', I will often write '$\forall x\ B$', '$\exists x\ B$' for any $B \to o$ to make quantified formulas easier to read.

The *general semantic schema* involving the *meaning* (i.e., a construction) of an expression $E$, *denotation* (i.e., the object, if any, denoted by $E$) and *reference* (i.e., the value of an intension, if the denotation is an intension, in the actual world at the present time) is depicted by Fig. 1.

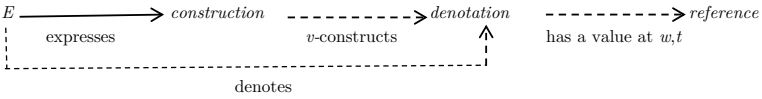


Fig. 1. TIL General semantic schema

Once the meaning construction of a term or expression has been given, it can be derived what the construction produces (if anything), i.e. what the denotation of $E$ is. Provided the denotation is not a trivial (i.e., constant) intension or a mathematical function, the reference cannot be logically derived; instead, it must be established by extra-logical and extra-semantic means (i.e., empirical inquiry or mathematical calculation) what the reference, if any, is.

As mentioned above, TIL is a logic of partial *functions*. Therefore, sets and relations are modelled by their characteristic functions. For instance, $(o\tau)$ is the type of a set of numbers, while $(o\tau\tau)$ is the type of a binary relation-in-extension between numbers. That an element $v$-constructed by $a \to \iota$ belongs to a set $M \to (o\iota)$, which in set-theoretical notation is written as '$a \in M$', is in TIL recorded as an application of the function $M$ to $a$: $[M\ a]$. For instance, having the set of prime numbers $Prime/(o\tau)$, the sentence "2 is a prime number" is furnished with this simple construction as its meaning: $[{}^0Prime\ {}^02]$.

Note that any non-procedural entities must be supplied to molecular constructions by Trivialization (or a variable, as the case may be). The reason is this. Parts or constituents of procedures can be only their (sub)procedures. No non-procedural abstract or concrete object can be a constituent part of a procedure. The objects on which procedures operate are beyond them. Thus, while *John* is an individual that cannot be executed and thus cannot be a part of a procedure, ${}^0John$ is a procedure, albeit trivial.[10]

Properties of individuals are intensions, objects of type $(o\iota)_{\tau\omega}$. In order to apply a property to an individual, a functional application is used. However, properties are not type-theoretically proper entities to be directly applied to an individual. They have to be extensionalized first. For instance, the sentence

"John is a surgeon"

ascribes the property of being a surgeon to John. As with any other non-procedural objects to be operated on, the individual John, as well as the property of being a surgeon, are supplied by their Trivialisation, ${}^0John$, ${}^0Surgeon$. Since the property is an intension of type $(((o\iota)\tau)\omega)$, or $(o\iota)_{\tau\omega}$ for short, the property must be applied to a possible world (type $\omega$) first and then to time (type $\tau$). To this end, we have variables $w \to \omega$ and $t \to \tau$; thus, we get $[[{}^0Surgeon\ w]\ t]$, or ${}^0Surgeon_{wt}$, for short. In this way, we obtain the population of surgeons in the world $w$ and time $t$, in which we are going

[10] In this paper, I do not deal with the semantics of proper names. Whenever used here, a proper name simply stands for a label of an individual. For the viewpoint on the TIL semantics of proper names, see Jespersen & Zouhar (1999) or Zouhar (2000).

to evaluate the truth value of the sentence. That John belongs to this population is expressed simply by the application of this population to John: $[{}^0Surgeon_{wt}\ {}^0John] \to \mathrm{o}$. Finally, we abstract over the values of the variables $w$ and $t$ to obtain the proposition that John is a surgeon.

$$\lambda w \lambda t\ [{}^0Surgeon_{wt}\ {}^0John] \to o_{\tau\omega}$$

So much for the basic technicalities of TIL.

Other ingredients that I need to illustrate the communication of agents, their reasoning and learning by messaging are the notions of requisite and refinement. (Duží et al. 2010, Ch. 4) introduces a logic of intensions that has been developed into an *intensional essentialism* which spells out how some intensions supervene on other intensions.[11] The key notion is that of *requisite.* Intuitively, a requisite of an intension $A$ is a further intension $B$ that must, as a matter of analytic necessity, be possessed by any entity that happens to be in the extension of $A$. For instance, the property of being unmarried is a requisite for having the initial property of being a bachelor; if an individual $a$ happens to be a bachelor, then it must be unmarried. Formally, a requisite is a relation-in-extension between intensions of any type, though typically between individual properties or offices. For the sake of simplicity, here I define the relation of requisite between individual properties of type $(\mathrm{o}\iota)_{\tau\omega}$. Since TIL is a logic of partial function, to deal with partiality properly, we need to apply the property $True/(\mathrm{o}\mathrm{o}_{\tau\omega})_{\tau\omega}$ of propositions. The reason is this. Propositions can have *truth-value gaps* in some worlds and times; in such a case, the extensionalisation of the proposition $P$, i.e. $P_{wt}$, fails to produce a truth-value, the Composition is $v$-improper. Partiality, as we all know very well, brings about technical complications. To deal with them, we define three properties of propositions *True*, *False* and *Undefined*, all of type $(\mathrm{o}\mathrm{o}_{\tau\omega})_{\tau\omega}$, as follows ($P \to \mathrm{o}_{\tau\omega}$):

$[{}^0True_{wt}\ P]$ $v$-constructs $\mathbf{T}$ if $P_{wt}$ $v$-constructs $\mathbf{T}$, otherwise $\mathbf{F}$;

$[{}^0False_{wt}\ P]$ $v$-constructs $\mathbf{T}$ if $\neg P_{wt}$ $v$-constructs $\mathbf{T}$, otherwise $\mathbf{F}$;

$[{}^0Undefined_{wt}\ P] = \neg[{}^0True_{wt}\ P] \wedge \neg[{}^0False_{wt}\ P]$.

[11] Intensional essentialism obtains between intensions, unlike individual anti-essentialism that concerns bare individuals.

**Definition 3** (*requisite*). Let $f$, $g \to (o\iota)_{\tau\omega}$ be constructions $v$-constructing properties; $True/(oo_{\tau\omega})_{\tau\omega}$ the property of a proposition of being true in a given world $w$ and time $t$; $x \to \iota$; $Req/(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$. Then the *property* $v$-constructed by $f$ is *a requisite of the property* $v$-constructed by $g$ iff

$$[^0Req\ f\ g] = \forall w \forall t\ \forall x\ [[^0True_{wt}\ \lambda w \lambda t\ [g_{wt}\ x]] \supset [^0True_{wt}\ \lambda w \lambda t\ [f_{wt}\ x]]].$$

*Remark.* This definition applies the property *True* to a proposition because the relation obtains necessarily.[12] If we carelessly defined the relation by way of $\forall w \forall t\ \forall x\ [[g_{wt}\ x] \supset [f_{wt}\ x]]$, the result would be a falsehood. The reason is that, at those worlds and times at which the Composition $[g_{wt}\ x]$ or $[f_{wt}\ x]$ is *v-improper*, the universal quantifiers would return the truth value F.

The property of propositions *True* is also applied in the definition of the difference between a *presupposition* and *mere entailment*.

**Definition 4** (presupposition vs mere entailment)
Let $P$, $Q$ be constructions of propositions. Then

*Q is entailed by P* iff

$\forall w \forall t\ [[^0True_{wt}\ P] \supset [^0True_{wt}\ Q]]$;

*Q is a presupposition of P* iff

$\forall w \forall t\ [[[^0True_{wt}\ P] \vee [^0False_{wt}\ P]] \supset [^0True_{wt}\ Q]]$.

As a *corollary*, we have:

*Q is a presupposition of P* iff

$\forall w \forall t\ [\neg[^0True_{wt}\ Q] \supset [^0Undefined_{wt}\ P]]$.

If a presupposition of a proposition $P$ is not true, then $P$ has no truth value.

[12] Indeed, the requisite relation obtains by analytical necessity, in all possible worlds. In artificial intelligence, a weaker condition is sometimes applied; then it means 'typically'. These typical properties related to an initial property are usually defined by means of defaults; for instance, the typical property of a bird is flying, unless it is a penguin or ostrich. For details, see Duží, Číhalová and Menšík (2011).

The relation of *refinement* obtains between *concepts*, i.e. closed constructions in their normal form.[13] Usually, we need to refine an atomic concept, i.e. Trivialisation of an entity. For instance, the atomic concept of the property of being a bachelor is ${}^0Bachelor$. Its refinement is an *ontological definition* of this property, where ontological definition is a molecular construction of the same property, like, for example

$\lambda w \lambda t\ \lambda x\ [[{}^0Unmarried\ {}^0Man]_{wt}\ x]$.

**Definition 5** (*refinement of a construction*) Let $C_1$, $C_2$, $C_3$ be constructions. Let ${}^0X$ be an atomic concept of $X$, and let ${}^0X$ occur as a constituent of $C_1$. If $C_2$ differs from $C_1$ only by containing in lieu of ${}^0X$ an ontological definition of $X$, then $C_2$ is a *refinement of* $C_1$. If $C_3$ is a refinement of $C_2$ and $C_2$ is a refinement of $C_1$, then $C_3$ is a *refinement of* $C_1$.

For the needs of agents' communication, we introduce the function-in-intension $Refine/(*_n*_m)_{\tau\omega}$ assigning to a construction/concept its refinement; $[{}^0Refine_{wt}\ {}^0C] = {}^0D$ means that the construction $D$ is a refinement of the construction $C$. Note that here we make use of the *hyperintensional* features of TIL. Constructions $C$ and $D$ do not occur in the execution mode; their products are irrelevant here. Rather, they are *presented* as arguments of the function *Refine*. Therefore, they must be supplied by Trivialization.

## 3. Different kinds of Wh-questions

*Empirical* questions denote non-constant $\alpha$-*intensions* of type $\alpha_{\tau\omega}$ that is functions with the domain of possible worlds. The direct answer to such a question is the value of type $\alpha$ of this intension in the actual world $w$ and

[13] Concept and the normal form of a construction are rigorously defined in (Duží et al. 2010, §2.2.1). Briefly, the normal form of a construction $C$ is the representant of the class of constructions that are procedurally isomorphic with $C$. It is defined as the alphabetically first, non-$\eta$-reducible construction.

time $t$ of evaluation.[14] Hence, the type of a possible direct answer dictates the type of content of an empirical question.

Empirical Yes-No questions denote *propositions* of type $\mathrm{o}_{\tau\omega}$, where $\mathrm{o}$ is the type of truth values.[15] The inquirer wants to know the truth-value of the proposition in question in the world $w$ and time $t$ of evaluation. For instance, the answer to the question "Is John a surgeon?" is Yes/No according as the proposition that John is a surgeon is true in $w$ and $t$. On the other hand, the variety of possible answers to Wh-questions is much greater depending on the type $\alpha$ of an $\alpha$-intension the value of which is asked for. For instance, one can ask for the value of an *individual office* (or *role*) of type $\iota_{\tau\omega}$, like "Which is the highest mountain in Slovakia?", "Who is the mayor of the city of Dunedin?", "Who is the No.1 player in ATP tennis singles"? A possible direct answer to such a question is a unique individual (an object of the type $\iota$) who happens to play a given role. For instance, the meaning of the question "Who is the mayor of the city of Dunedin?" comes down to this construction.

$$\lambda w \lambda t\, [{}^0\mathrm{I}\ \lambda who\ [who = [{}^0Mayer\text{-}of_{wt}\ {}^0Dunedin]]] \rightarrow \iota_{\tau\omega}$$

Types. $\mathrm{I}/(\iota(\mathrm{o}\iota))$: the singularizer, i.e. the function that associates a set $S$ of individuals with the only member of $S$ provided $S$ is a singleton, and otherwise (if $S$ is an empty or a multi-valued set) the function I is undefined; $who \rightarrow \iota$: the variable ranging over individuals such that the individual plays the role of the Mayor of Dunedin in the world $w$ and time $t$ of evaluation (the direct answer should be provided by the valuation of this variable); $Mayer\text{-}of/(\iota\iota)_{\tau\omega}$: an attribute, i.e. an empirical function that associates a given individual with another individual (in this case that one who is a Mayer of something); $Dunedin\ /\iota$.

[14] (Duží, Číhalová 2015) distinguishes between *direct* and *complete* answer to an empirical question. *Direct* answer is an object $X$ of type $\alpha$ that is the value (in the world and time of evaluation) of the $\alpha$-intension asked for, while *complete answer* is the proposition that the value of the asked intension is the object $X$. The authors deal with presuppositions of questions. Their main thesis is this. If a presupposition of a given question is not true, then there is no direct answer. Instead, a plausible complete answer is the negated presupposition.

[15] For details on TIL analysis of questions and answers see (Duží et al. 2010, §3.6.).

Note that the question transforms into a construction of an individual office, as it should be. The agent would like to know the value of this office.

Another frequent type of intensions is the *property of individuals*, an object of type $(o\iota)_{\tau\omega}$. For instance, the direct answer to the question "Which are the private hospitals located in Lowestoft?" should convey a set (of type $(o\iota)$) of individuals. There are two kinds of possible direct answers. An *exhaustive answer* conveys a complete list of individuals with the property of being a private hospital in Lowestoft, while an incomplete answer provides just some of them. Anyway, in both cases, the answer should be *conclusive*; it means that the individuals belonging to this list should be referred to directly. An indirect description of an individual would not be satisfactory.[16] For instance, the answer "They are the private hospitals located in the most eastern city of England" is not conclusive. The agent would have to go on asking, "Which is the most eastern city of England?" and "Which are the private hospitals in the most eastern city of England?" and so on.

Thus, the exhaustive answer to the question would be, for instance, the set: {Carlton Court, Airey Close, Beccles Hospital Inpatients, East Point Consulting Rooms, Andaman Surgery, James Paget Hospital, East Coast Community, The Veterinary Surgery, Crest View Medical Centre}.
The analysis of the question that constructs a property of individuals (that are asked for) is this.

$$\lambda w \lambda t\, [\lambda x\, [[[{}^0\mathit{Private}\ {}^0\mathit{Hospital}]_{wt}\, x] \wedge [{}^0\mathit{Located\text{-}in}_{wt}\, x\ {}^0\mathit{Lowestoft}]]] \rightarrow (o\iota)_{\tau\omega}$$

Types. $x \rightarrow \iota$: the variable ranging over individuals; *Private*/$((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$: property modifier: an analytic function that assigns to a property another (modified) property;[17] *Hospital*/$(o\iota)_{\tau\omega}$; *Located-in*/$(o\iota\iota)_{\tau\omega}$; *Lowestoft*/$\iota$.

One can also ask for the value of an attribute at an argument like the salary of somebody. The possible answer to the question "What is John's salary?" is a number, and the question denotes a magnitude of type $\tau_{\tau\omega}$.

[16] This problem has been dealt with in Duží (2022).

[17] The analyses of property modifiers has been introduced in Jespersen, Carrara, Duží (2017) or in Duží (2017).

### *3.1 Classification of Wh-questions*

Číhalová & Duží (2022) introduce the classification of Wh-questions based on the type of a possible answer. They show that for our purpose, the linguistic classifications are too coarse-grained and non-plausibly oriented. For the needs of a multi-agent system, we classify questions not only from the linguistic point of view but also from the logical point of view, with respect to a domain of interest and the structure of the agent's knowledge base. The authors distinguish between *static entities,* like necessary relations between properties of individuals and *dynamic entities,* like activities which form processes. Active actions and passive events are *activities*. Each activity can involve other objects that are called their *participants*.

The specification of *activities* is based on the linguistic theory of *verb valency frames.*[18] From the logical point of view, we deal with the verb phrases as denoting a *function* that is applied to its arguments. The number of arguments is controlled by the content verb *valency*. There are several types of valency. An *impersonal* (avalent) verb has no subject or a dummy subject. "It rains." is a typical example. Here the grammatic subject 'it' is just a dummy subject because it does not refer to any concrete object.[19] An *intransitive* (monovalent) verb has just one argument, the *subject* S; "John

[18] For the linguistic theory of verb valency frames, see Horák (1998) or Rambousek, Hlaváčková (2011). Číhalová (2016) proposed ontology of events based on the theory of verb valency frames. This theory is not unlike Chomsky's θ-theory, which is concerned with the distribution and assignment of thematic roles to arguments. The theta criterion describes the specific match between arguments and thematic roles in the logical form of a sentence. (I am grateful to the anonymous reviewer for drawing my attention to this theory.) Yet, since our research is a part of a broader project on linguistic and logical natural language analysis and processing, and since in this project we cooperate with the centre for computational linguistics in Masaryk University of Brno, we vote for the theory of verb valency frames. This theory is supported by the centre, where the lexicon of verb valencies (VerbaLex) has been developed.

[19] Lots of languages, including Romance and Slavonic ones, drop the dummy subject ('it', 'es', …) altogether, and make sentences just with a verb in third person singular.

(S) is singing." A *transitive* (divalent) verb has two arguments, an agent (A) and a patient (P), as in "John (A) kicked the ball (P)." A *ditransitive verb* has three arguments, an agent and two patients, for instance, in "John (A) passed the ball (P) to Tom (P)." There are also a few verbs with more than three arguments (polyvalent, like tritransitive); yet they mostly arise by valency increasing, where causatives or applicatives are typical valency increasing devices.[20]

Verb valency frames determine the obligatory and facultative arguments, i.e. thematic roles of a given verb, together with their types. Facultative arguments can be missing, of course. For instance, the verb 'buy' can occur in several sentences with a different number of arguments like "Tom bought a book", "Tom bought a book in Paris", "On Friday, Tom bought a book", "Tom bought a book for Jane in Paris", etc. In our analysis, we have to take these varieties into account. Linguists have created many classifications based on verb valency frames, for instance, VALLEX or VerbaLex.[21]

John Sowa (2000) proposed a specification tool for knowledge representation, where he adopted a linguistic approach to verbs. He developed the system of conceptual graphs in which Peirce's logic is combined with the semantic networks known from artificial intelligence. For the valency participants, Sowa uses the term '*thematic roles*' or '*case relations.*' His summary of all the thematic roles can be found in (Sowa 2000, pp. 506-510) or in the web source *Thematic roles*. Sowa distinguishes several types of thematic roles, for instance, Agent, Beneficiary, Destination, Duration, Effector, Experiencer, Instrument, Location, Matter, Patient and so on.[22] Thematic role or the type of a participant expresses the role that a noun phrase plays for the activity described by a governing verb. From the viewpoint of logic, it is the relation between two entities where one is an activity (expressed by the verb), and the other is an attribute (expressed mostly by a noun, adverb, number or adjective).

The number and the categories of participants depend on the respective domain of interest and the functions of the system of agents. In this paper,

[20] For details, see Dixon (2000).

[21] See, for instance Lopatková et al. (2006) and Hlaváčková, Horák (2006).

[22] For details, see Sowa (2000, 508-510).

I will use the following frequent kinds of attributes that can be assigned to an activity:

Pat – object affected by the activity

Ben – beneficient (somebody who has benefited from the activity)

Man – the manner of the activity execution (measure, speed etc.)

Inst – instrument

Time – when

Loc – the place of activity

Dir1 – the direction of activity – *from where*

Dir2 – the direction of activity – *which way*

Dir3 – the direction of activity – *where to*

Wh-questions concern the participants of activities; we ask for their values in a world and time of evaluation. Hence, we can distinguish questions about the process itself (*what* is going on?) from Wh-questions on the primary agent and other participants of a given activity. For instance, assume we have the sentence "John (the agent) is going (the activity) to London (Dir3) by car (Inst) in an average speed of 50 miles per hour (Man)." Then we can ask, "What is John doing?", "Who is going to London?", "How quickly does John go to London?" etc.

### *3.2 Hyperintensional questions about concepts*

A particular category of questions concerns hyperintensional questions about a given *concept.* The agents should be able to *learn* from experience through mutual communication with their fellow agents. In such a communication, it may happen that a receiving agent $b$ does not 'know' a concept that is a constituent of a sender's message. By 'knowing a concept' $C$, we mean having the concept $C$ in one's ontology. In such a situation, the receiving agent $b$ can ask for an explication or a definition of the unknown concept. When asking for the explication of concept $C$ the agent does not talk about the object produced by $C$. Rather, the concept, i.e. the closed

construction $C$ itself, is a subject matter that is asked for. Such a context where the construction $C$ is just *presented* as an argument rather than *executed* to produce an object is *hyperintensional.* In (Duží & Vojtáš 2008), a special kind of question is introduced, namely a question with the performative *Unrecognized*, the argument of which is an unknown concept $C$. The answer is then of type *Refine*, where the message provides a concept $C'$, which refines the unknown concept $C$.

*Refinement* has been rigorously defined above (Def.5). Briefly, by refining an atomic concept of an object $O$, we mean discovering a molecular concept that produces the same object $O$. In mathematics, refining usually concerns definitions like "a *group* is a set $G$ equipped with a binary operation that combines any two elements of $G$ to form another element of $G$ in such a way that group axioms are satisfied, namely associativity, the existence of the neutral element in $G$ and invertibility." Here the atomic concept to be refined is that of a 'group'. The molecular concept refining 'group' is encoded by the definiens, namely 'a set $G$ equipped with a binary operation that combines any two elements of $G$ to form another element of $G$ in such a way that group axioms are satisfied, namely associativity, the existence of the neutral element in $G$ and invertibility'. In the case of *empirical* concepts, it is more plausible to speak about *explication*. The reason is this. To say that a molecular concept $C$ is a refinement of an atomic empirical concept $D$ is risky. It would be a refinement only if the molecular concept $C$ were *analytically equivalent* to the original concept $D$, which means that both are the concepts of the same object $O/\alpha_{\tau\omega}$. However, in the most interesting cases of *empirical* concepts of PWS-*intensions* we use a Carnapian *explication* rather than a definition proper. Then equivalence is undoubtedly not guaranteed, for one can hardly check the identity of the intensions produced by the two concepts. Rather, a new molecular concept $C$ (explicatum) should define an intensional object $O$ that is as close as possible to the object referred to by an inexact (prescientific) concept $D$ (explicandum).

In *Meaning and Necessity* (1947), Carnap characterises explication as follows:

> The task of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather of replacing it by a newly constructed,

> more exact concept, belongs among the most important tasks of logical analysis and logical construction. We call this the task of explicating, or of giving an *explication* for, the earlier concept [...] (Carnap 1947, pp. 7-8)

Keeping this difference in mind, I use the term 'refinement' for both cases, including the explication of empirical concepts. In most cases of explicating the concept unknown to an agent, this simplification is harmless.

## 4. Agents' dynamic activities

The basic idea of the analysis is due to (Tichý 1980). Its adjustment and simplification are introduced in (Duží 2010). Tichý draws a distinction between *episodic* and *attributive* verbs. Attributive verbs ascribe properties to individuals, and their structure is usually a copula followed by an adjective or noun; for instance, 'is happy', 'is red', 'looks speedy', 'is a student' are attributive verbs. On the other hand, episodic verbs express actions performed by objects. For instance, if John is getting up, it would be insufficient to analyse this activity by assigning the property of getting up to John. Rather, John is *doing* the activity of getting up. For example, the sentence "*John is driving from Brussels to Paris at the average speed of 90 km/h*" should be analysed as describing a time-consuming *process* consisting of a series of *actions* and *events*. In (Číhalová, Štěpán 2014), the basic idea of specifying event ontology by means of verb valency frames was introduced, and (Číhalová, 2016) proposed its further adjustment. It consists, in particular, in refining the type of action executed within a given process. For instance, the specification of the process *Charles is driving from Prague to München by train at the speed of 90 km/h* is determined by the sense of the verb 'to drive' together with its arguments (*who* is driving – the actor, *when* is (s)he driving, *from where*, *to where*, by *what kind of a vehicle*, in *which speed,* etc.).

### *4.1 Agents' activities in the present*

From the logical point of view, an episodic verb denotes a relation-in-intension *Do* between an individual of type ι (the actor) and an activity.

Using a general placeholder α for the type of activity, *Do* thus obtains the type $(o\iota\alpha)_{\tau\omega}$.[23]

As mentioned above, each activity has several *participants* (i.e. assignments of an attribute to the activity), and the valency of the verb determines the compulsory participants and the maximal number of facultative participants. The attributes can be of various kinds like individuals, properties, quantities, etc. Typical kinds of attributes have been specified above. They are *Pat* (object affected by the activity), *Ben* (who has a benefit from the activity), *Manner* (manner of the activity execution), *Inst* (instrument), *Time* (when), *Time1* (time when the activity started), *Time2* (time when the activity ended), *Loc* (location of the activity), *Dir1* (direction of event – from where), *Dir2* (direction of event – where through), *Dir3* (direction of event – where to). If needed, other kinds of attributes can be specified. For the purpose of the system implementation, we only must keep the selected keywords fixed.

The type of assigning an attribute to an activity is the relation in intension between an object of type β and the activity (type α); where β can be a property of individuals like being a train, or a number of type τ (time), individual ι (like John, Prague, Brussels) etc., according to the kind of an attribute. Thus, we have a general type of participant *Part*/$(o\beta\alpha)_{\tau\omega}$. It must be a relation-*in-intension*, as one and the same activity can be performed with different instruments at different times, and so like. For instance, John can go from Prague to Brussels by train, and next time he can vote for a plane.

[23] In this paper, I often release typing and use instead placeholders like α, β, δ for entities too complicated from the typing point of view. As we all know well, typed languages and calculi are useful and easy to work with because typing prevents a user from making silly mistakes when specifying procedures. Yet, too strong typing can sometimes be restrictive. For this reason, typed functional programming languages are usually polymorphic, or type control is not too strict; in case of a typing error, the interpreter only informs the programmer and leaves the decision to them. As TIL is a typed lambda calculus, in its computational variant TIL-Script, we also aim to implement such useful features. Proposals of the polymorphic TIL system have been introduced in Duzi (1993), Pezlar (2020) and Pezlar (2022). For a benevolent type checking algorithm, see, e.g., Duží,Marie & Fait,Marie (2019).

A *general pattern* for the analysis of an activity $P \to \alpha$ with the actor $A \to \iota$ and participants *Part-i*/$(o\beta\alpha)_{\tau\omega}$ that assign attributes $X_i \to \beta_i$ to $P$ is this:[24]

$$\lambda w \lambda t\, [[^0Do_{wt}\, A\ P] \wedge [^0Part\text{-}1_{wt}\ X_1\, P] \wedge$$
$$[^0Part\text{-}2_{wt}\ X_2\, P] \wedge \ldots \wedge [^0Part\text{-}n_{wt}\ X_n\, P]]$$

For instance, the analysis of the sentence "*John goes to Brussels by train*" comes down to this construction.

$$\lambda w \lambda t\, [[^0Do_{wt}\, ^0John\ ^0Go] \wedge [^0Inst_{wt}\, ^0Train\ ^0Go] \wedge$$
$$[^0Dir3_{wt}\, ^0Brussels\ ^0Go]]$$

It may happen that at another time John will go to Brussels by plane. Then we have

$$\lambda w \lambda t\, [[^0Do_{wt}\, ^0John\ ^0Go] \wedge$$
$$[^0Inst_{wt}\, ^0Plane\ ^0Go] \wedge [^0Dir3_{wt}\, ^0Brussels\ ^0Go]]$$

Wh-questions about John's activity would be, for instance: *What* does John do? *Where* does John go? The content of these questions transforms into constructions like (variables *what* $\to \alpha$, *where* $\to \iota$)

$$\lambda w \lambda t\ \lambda what\ [^0Do_{wt}\, ^0John\ what]$$
$$\lambda w \lambda t\ \lambda where\ [[^0Do_{wt}\, ^0John\ ^0Go] \wedge [^0Dir3_{wt}\ where\ ^0Go]]$$

The technique of deducing answers to such Wh-questions has been introduced in Duží, Fait (2020) and (2021). It is an adjusted system of natural deduction with special rules rooted in the rich semantics of natural language and some technical TIL rules stemming from the need to work within a hyperintensional context. Classical natural deduction rules can be applied only to constituents of a construction. For this reason, we need these special

[24] The first proposal of such an analysis of activities with participants has been introduced in Duží (2021). In this paragraph, I introduce a slightly adjusted and corrected analysis. In particular, I do not apply the relation-in-intension *Assign* (an attribute to an activity), as this entity is superfluous and we can obtain a more elegant solution without it.

technical rules.[25] In principle, answers to such Wh-questions are derived by unifying matching terms by means of substituting the values for variables like *what*, *where*, and so like. In our simple example, the answers would be $what = {}^0Go$, $where = {}^0Brussels$.

If agent $b$ has in his ontology the specification of all the possible participants of an activity, and if $b$ obtains an incomplete message where some participants are missing, then $b$ can ask his fellow agents to complete the missing pieces of knowledge. For instance, when receiving the first message about John's going to Brussels by train, the agent can send another query message asking from *where* does John go to Brussels. To this end, we apply the method of analysis of Wh-questions, as introduced above. The content of the query is then this.

$$\lambda w \lambda t\, \lambda d\, [[{}^0Do_{wt}\, {}^0John\, {}^0Go] \wedge \\ [{}^0Inst_{wt}\, {}^0Train\, {}^0Go] \wedge [{}^0Dir1_{wt}\, d\, {}^0Go] \wedge [{}^0Dir3_{wt}\, {}^0Brussels\, {}^0Go]]$$

A possible answer to this Wh-question is the message with this content.

$$\lambda w \lambda t\, [[{}^0Do_{wt}\, {}^0John\, {}^0Go] \wedge [{}^0Inst_{wt}\, {}^0Train\, {}^0Go] \wedge \\ [{}^0Dir1_{wt}\, {}^0Prague\, {}^0Go] \wedge [{}^0Dir3_{wt}\, {}^0Brussels\, {}^0Go]]$$

The answer is obtained by substituting Prague for the variable $d$ using the agents' knowledge base.[26] In case there are two or more actors of the activity, we can apply the relation-in-intension $Do'/(\mathrm{o}(\mathrm{o}\iota)\alpha)_{\tau\omega}$. For instance, the sentence "*John and Tom go to Brussels by plane on April 1st*" is furnished with this analysis.

$$\lambda w \lambda t\, [[{}^0Do'_{wt}\, \lambda x\, [[x = {}^0John] \vee [x = {}^0Tom]]\, {}^0Go] \wedge \\ [{}^0Inst_{wt}\, {}^0Plane\, {}^0Go] \wedge [{}^0Dir3_{wt}\, {}^0Brussels\, {}^0Go] \wedge \\ [{}^0Time_{wt}\, {}^0April1\, {}^0Go]]$$

The above sentence is underspecified, as it is not clear whether John and Tom are going on their own or together. Yet, the analysis is unambiguous,

[25] See, for instance, Duží,Marie, Jespersen, B. (2015) and Jespersen, B., Duží,Marie (2022), where the rules for existential quantification into hyperintensional contexts have been introduced.

[26] For details on deducing answers to Wh-questions by applying the system of natural deduction adjusted to TIL, see Duží, Fait (2021).

as John and Tom are the two actors of *the same activity.* Hence, they are going together. If they went each on their own, it would be two different activities with different actors, even if the other participants were identical.[27]

### *4.2 Agents' activities in past or future*

Another advantage of this approach is this. Since in TIL, we have two modal parameters, time and possible worlds, we can easily specify activities executed in *past* or *future* and model the *dynamic behaviour* and reasoning of agents. If an activity was executed in the past or will be executed in future, the sentence should contain a reference to the *time* when this or that happened or will happen. For instance, the sentence "*John will go to Brussels by plane*" receives this analysis.

$$\lambda w \lambda t\, \exists t' \; [[{}^0Do_{wt'}\, {}^0John\; {}^0Go] \wedge [t' > t] \wedge$$
$$[{}^0Inst_{wt}\; {}^0Plane\; {}^0Go] \wedge [{}^0Dir3_{wt}\; {}^0Brussels\; {}^0Go]]$$

Note that the attributes *Inst* and *Dir3* are extensionalised with respect to time $t$ of evaluation rather than to time $t' > t$, as we assign these attributes now. The situation can change; of course, John can later vote for a car, for instance. In such a case, the sentence is not true.

Anyway, the piece of information conveyed by the sentence seems to be incomplete, as one is tempted to ask, "*When* will John go to Brussels?" It is so because sentences in the past or future should contain a constituent referring to *time* $T \to (o\tau)$, the time interval when this or that happened or will happen. In such a case, the sentence is associated with a *presupposition* that the current time $t$ is in the proper relation with respect to $T$. Roughly, it means that for sentences in future, $t$ comes before the end of the reference time $T$, while for sentences in past, $t$ comes after $T$; if it is not so, then the proposition denoted by the sentence has a truth-value gap. For instance, the sentence "John will go to Brussels on January 1st, 2023" can be true or false till January 1st, 2023, 24:00. Later, it has no truth value. Involving presupposition is reasonable, of course. Imagine a situation when

[27] I am grateful to the anonymous reviewer for this remark, which lead me to the specification of an activity that is not ambiguous.

(a) asks, "Shall we meet today at 5 p.m.?" using an SMS message, and (b) reads the message later than 5 p.m. Then (b) cannot answer Yes or No. Instead, (b) answers by negating the presupposition, e.g., "Sorry, it is later than 5 p.m. now".

In English, simple past and present perfect are distinct tenses, and we should be able to differentiate them (similarly for simple future and future perfect tenses). While the simple past tense is used for the activities in past that have been finished in past, the present perfect tense is used for past actions that are related to or continue into the present. Detailed analysis of sentences in present perfect tense can be found in Tichý (1980) or Duží et al. (2010, 2.5.2). Briefly, using simple past, the time $t$ of evaluation must be greater than the end of the reference time interval $T$, while for present perfect $t$ must be greater than the beginning of this interval.

Moreover, the sentence can also convey information on the *frequency* of the activity to be executed in the reference time $T$ like 'twice', 'always', 'all the time since', 'for the whole year'. Tichý (1980) introduces a detailed analysis of such sentences in all English tenses. Tichý's analysis is difficult to understand because Tichý applies the *singulariser* function to a singleton typed as containing a truth value in order to make the set fail to deliver a truth value in case the associated presupposition is not satisfied.[28] Tichý's analysis is analogous to what the computer scientist would call an *imperative* rather than a *declarative* analysis. The downside to an imperative analysis is that it may conceal flaws that rear their head only when the analysis is applied to extreme situations. Yet there is an elegant alternative that uses the 'if-then-else' connective proposed by Duží (2010).[29] The author demonstrates here the method of a fine-grained analysis of such sentences equivalent to Tichý's approach but easier to read. In the paper, a general analytic schema for sentences that come associated with a presupposition is presented. To this end, a strict definition of the *If-then-else-fail* function that complies with the compositionality constraint is utilised. In this paper, I am going to apply this solution. Summarising briefly, consider a sentence $S$ with a presupposition $P$. It encodes a meaning procedure, the evaluation of which can be described as follows:

[28] The same method is reproduced in Duží et al. (2010, 2.5.2).

[29] See also Duží (2019b).

> In any ⟨*w, t*⟩-pair of evaluation, *if* $P_{wt}$ is true *then* evaluate $S_{wt}$ to produce a truth value, *else fail* to produce a truth value.

To formulate the schema rigorously, we need to define the *if-then-else-fail* function. First, we define the *if-then-else* function. Here is how. The procedure encoded by "If $P$ ($\to$ o) then $C$ ($\to \alpha$), else $D$ ($\to \alpha$)" is a two-phase procedure that produces a (strict) function of type $(\alpha o *_n *_n)$. Its definition decomposes into two phases.

*First*, select a construction to be executed based on a specific condition $P$. The choice between $C$ and $D$ is specified by this Composition:

$$[^0\urcorner^* \lambda c\, [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]]$$

Types: $P \to o$ $v$-constructs the condition of choice between the execution of $C$ or $D$, $C \to *_n$, $D \to *_n$, $^2C$, $^2D \to \alpha$; $c \to *_n$; $\urcorner^*/(*_n (o *_n))$: the singularizer function that associates a singleton of constructions with the only element of this singleton, and is otherwise (i.e. if the set is empty or many-valued) undefined.

If $P$ $v$-constructs **T** then the variable $c$ $v$-constructs the *construction* $C$, and if $P$ $v$-constructs **F** then the variable $c$ $v$-constructs the *construction* $D$. In either case, the set constructed by

$$\lambda c\, [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]$$

is a singleton and the singularizer $\urcorner^*$ returns as its value either the construction $C$ or the construction $D$.[30]

*Second*, the selected construction is executed; therefore, Double Execution must be applied:

$$^2[^0\urcorner^* \lambda c\, [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]]$$

[30] Note that in this phase $C$ and $D$ are not constituents to be executed; rather they are merely supplied as objects to be selected by the variable $c$. This is to say that in TIL constructions themselves can be objects to be operated on, and without this *hyperintensional* approach we would not be able to define the *strict* function *if-then-else*. For the difference between constructions occurring in the *displayed* and *executed mode*, see, for instance, Duží (2019).

As a special case of the *if-then-else-fail* function, *no* construction $D$ is to be selected whenever $P$ is not satisfied. Thus, the definition of the *if-then-else-fail* function of type $(\alpha o{*_n})$ is this:

$$^2[^0\urcorner{*}\ \lambda c\ [P \wedge [c = {}^0C]]]$$

Indeed, if $P$ *v*-constructs $\mathbf{F}$, then the class constructed by $\lambda c\ [P \wedge [c = {}^0C]]$ is empty so that the singularizer function does not return as its value any construction. As a result, according to Def. 1, both the composition $[^0\urcorner{*}\ \lambda c\ [P \wedge [c = {}^0C]]]$ and its Double Execution are *v*-improper. Applying this definition to the case of an empirical presupposition, we obtain this. Let $P/{*_n} \rightarrow o_{\tau\omega}$ be a construction of a presupposition of $S/{*_n} \rightarrow o_{\tau\omega}$. Furthermore, let $c/{*_{n+1}} \rightarrow {*_n}$, $^2c \rightarrow o$. Then the type of the *if-then-else-fail* function is $(oo{*_n})$ and its definition comes down to this construction:

$$\lambda w\lambda t\ [^0\textit{if-then-else-fail}\ P_{wt}\ {}^0[S_{wt}]] = \lambda w\lambda t\ {}^2[^0\urcorner^{*}\ \lambda c\ [P_{wt} \wedge [c = {}^0[S_{wt}]]]]$$

Instead of the above definition, I use the abbreviated notation to make the *general analytic schema* easier to read:

$$\lambda w\lambda t\ [\textit{if}\ P_{wt}\ \textit{then}\ S_{wt}\ \textit{else fail}].$$

For instance, the truth conditions of the sentence "*John will go to Brussels by plane in 2023*" presuppose that the current time $t$ in which the truth conditions are being evaluated comes before 2023. In other words, the year 2023 comes in future with respect to time *t*. If it is not so, the sentence has *no truth value*. Thus, we have

$$\lambda w\lambda t\ [\textit{If}\ [^0\textit{Future}_t\ {}^0\textit{2023}]\ \textit{then}\ [[\exists t'\ [^0\textit{Do}_{wt'}\ {}^0\textit{John}\ {}^0\textit{Go}] \wedge [^0\textit{2023}\ t']] \wedge [^0\textit{Inst}_{wt}\ {}^0\textit{Plain}\ {}^0\textit{Go}] \wedge [^0\textit{Dir3}_{wt}\ {}^0\textit{Brussels}\ {}^0\textit{Go}] \wedge [^0\textit{Time}_{wt}\ {}^0\textit{2023}\ {}^0\textit{Go}]]\ \textit{else fail}]$$

The analysis can also account for the frequency of the activity to be executed in the reference time interval $T$. The general analytic schema for sentences $S$ in future tenses is this.

$$\lambda w\lambda t\ [^0\textit{Future}_t\ [^0\textit{Frequency}_w\ S]\ {}^0\textit{In-Time}] =$$
$$\lambda w\lambda t\ [\textit{If}\ [^0\textit{In-Time} >_\tau t]\ \textit{then}\ [[^0\textit{Frequency}_w\ S]\ {}^0\textit{In-Time}]\ \textit{else fail}].$$

Here $>_\tau$ means that the reference interval *In-Time*/$(o\tau)$ comes after time $t$, *Future* receives the same type as *Past* (which is applied for sentences in

past tenses), that is $((o(o(o\tau))(o\tau))\tau)$; $S$ is the proposition to be evaluated and *Frequency* is the frequency of time intervals in which the proposition $S$ takes the truth-value **T** in world $w$. Hence, the modifier *Frequency* is of type $((o(o\tau))o_{\tau\omega})_\omega$. The schema for sentences in past tenses is similar; it differs only by applying the constituent *Past* instead of *Future.*[31]

If John's activity of going to Brussels by plane in 2023 will be twice a month, by applying the above schema, we obtain this construction.

$$\lambda w \lambda t\, [\mathit{If}\ [{}^0\mathit{2023} >_\tau t]\ \mathit{then}\ [[{}^0\mathit{Twice\text{-}month}_w\, \lambda w \lambda t\, [[{}^0\mathit{Do}_{wt}\ {}^0\mathit{John}\ {}^0\mathit{Go}] \wedge [{}^0\mathit{Inst}_{wt}\ {}^0\mathit{Plain}\ {}^0\mathit{Go}] \wedge [{}^0\mathit{Dir3}_{wt}\ {}^0\mathit{Brussels}\ {}^0\mathit{Go}]]]\ {}^0\mathit{2023}]\ \mathit{else}\ \mathit{fail}]$$

Detailed analysis of *Frequency* can also be found in Duží et al. (2010, §2.5.2) or Duží (2010).

### *4.3 Agents' learning new concepts*

As mentioned above, agents can *learn* by experience. They are "born" with a minimal ontology of concepts, which is gradually extended during the agents' life cycle.[32] When agent $a$ receives a message from agent $b$ containing a concept $C$ not contained in $a$'s ontology, $a$ does not understand the message. In such a case, agent $a$ answers to $b$ by sending a query message asking for a *refinement* (i.e. a definition or explication utilising simpler concepts) of the unknown concept $C$. In this way, agents learn new concepts and share their knowledge.[33] To this end, we introduce two 'instructions over concepts', i.e. these relations-in-intension:

[31] A detailed analysis of particular kinds of tenses can be found in (Duží et al. 2010, §2.5.2).

[32] Concept is defined in TIL as a closed construction in its normal form. For details, see Duží et al. (2010, §2.2).

[33] Similar conception has been applied in (Číhalová et al. 2010). In Menšík et al. (2019), the authors introduce the method of refining or explicating atomic concepts by molecular ones using machine learning techniques adjusted to natural language processing. In this way, the agents can learn not only by asking their fellow agents, but also by exploring their environment, in particular by obtaining new pieces of knowledge from the huge amount of text data that are in our disposal.

*Unrecognized*/$(\text{o}*_n)_{\tau\omega}$: a property of a concept that an agent does not know it;

*Refine*/$(*_n*_n)_{\tau\omega}$: an empirical function that assigns to a concept $C$ another concept $D$ such that $D$ is a refinement of $C$.

To adduce an example, consider a short communication between agents (a) and (b):

(a) The Incan people used khipu for recording pieces of knowledge.

(b) I do not recognise khipu (I don't know what 'khipu' means, what does 'khipu' stand for.)

(a) Khipu is a recording device fashioned from knotted strings. It had been historically used by a number of cultures in the region of Andean South America, in particular by the Incan people, but also by the ancient Chinese, Tibetans and Japanese.

For the sake of simplicity, I analyse here only the first three sentences and ignore the last one 'It had been historically used by a number of cultures in the region of Andean South America, in particular by Incan people, but also by the ancient Chinese, Tibetans and Japanese.'.

In order to make the content of the agent's (a) first message clear and easier to analyse, let me slightly reformulate the sentence: "There were Incan people who did the activity of recording pieces of knowledge by means of khipu." Here we can use the simple past because the message does not contain any reference time when this activity used to be done, and it definitely stopped being done a long time ago. Thus, we obtain

$$\lambda w \lambda t\, \exists u\, [[u < t] \wedge \exists x\, [[[{}^0\mathit{Incan}\ {}^0\mathit{People}]_{wu}\, x] \wedge [{}^0\mathit{Do}_{wu}\, x\ {}^0\mathit{Record}]] \wedge [{}^0\mathit{Inst}_{wt}\ {}^0\mathit{Khipu}\ {}^0\mathit{Record}] \wedge [{}^0\mathit{Pat}_{wt}\ {}^0\mathit{Knowledge}\ {}^0\mathit{Record}]]$$

Types. $t,u \rightarrow \tau$; $x \rightarrow \iota$; *Incan*/$((\text{o}\iota)_{\tau\omega}(\text{o}\iota)_{\tau\omega})$: property modifier; *People, Khipu*/$(\text{o}\iota)_{\tau\omega}$; *Record(ing)*/$\alpha$: activity; *(pieces of) Knowledge*/$(\text{o}*_n)$.

Agent's (b) asking for a definition or refinement of the khipu concept is analysed simply as

$$\lambda w \lambda t\, [{}^0\mathit{Unrecognized}_{wt}\ {}^{00}\mathit{Khipu}]$$

Gloss. The atomic concept $^0$*Khipu* has not been recognised; therefore, this very construction must be supplied by another Trivialisation.

Finally, the content of the agent's (a) refining message comes down to this construction.

$$\lambda w \lambda t\, [^0\mathit{Refine}_{wt}\ ^{00}\mathit{Khipu}] = {}^0[\lambda w \lambda t\, \lambda x\, [[[^0\mathit{Recording}\ ^0\mathit{Device}]_{wt}\ x] \wedge \exists y\, [[[^0\mathit{Knotted}\ ^0\mathit{String}]_{wt}\ y] \wedge [^0\mathit{Fashioned\text{-}from}_{wt}\ x\ y]]]]$$

Additional types. $x, y \to \iota$; *Recording, Knotted*/$((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$: property modifiers; *Device, String*/$(o\iota)_{\tau\omega}$; *Fashioned-from*/$(o\iota\iota)_{\tau\omega}$.

Gloss. Again, a slightly reformulated but equivalent sentence is analysed above, namely this: *Khipu* is a property of individuals $x$ such that $x$ is a recording device and there are individuals $y$ such that they are knotted strings and $x$ is fashioned from $y$. It is harmless here not to seek a strictly literal analysis.

Note that here we again utilise *hyperintensional* features of TIL. The very *concept*, i.e., the *construction* of the respective entity, is asked for refining. An agent who is asking for refinement wants to obtain more detailed *instructions* so that they would understand the message. And this instruction, i.e. procedure, is an object to deal with here rather than the product of the procedure.

## 5. Conclusion

In this paper, we dealt with agents' dynamic activities specified in different tenses. To this end, the linguistic and logical analysis of Wh-questions has been utilised. After a brief introduction to the fundamentals of our background theory of Transparent Intensional Logic (TIL), the logical analysis of Wh-questions and answers in TIL has been illustrated by examples of agents' communication in TIL. Dynamic aspects of agents' reasoning, including messages on participants of activities specified in different tenses and agents' learning by messaging in mutual communication with their fellow agents, have been analysed and demonstrated by examples. The main novelty of the paper is a detailed analysis of agents' activities in present, past and future, specified with reference to the time when the activity

happened or will happen to be done together with the frequency of the activity in the reference time.

Further research will concentrate on a still more detailed analysis of messages in different grammatical tenses, presuppositions of such messages, and on dynamic aspects of agents' activities. Here we will also apply the results obtained in the application of Gentzen's natural deduction adjusted for TIL so that these methods can be integrated into one intelligent system.

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RESEARCH ARTICLE

# Actions, Products, Demonstrations

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*Abstract.* As it is broadly accepted, typical uses of demonstratives are accompanied by demonstrations. The concept of demonstration, however, manifests the action–product ambiguity analogous to that visible in the opposition between jumping and the resulting jump, talking and the resulting talk or crying and the resulting cry. It is also a heterogeneous concept that enables demonstrations to vary significantly. The present paper discusses action–product ambiguity as applied to demonstrations as well as the heterogeneity of the latter. An account that acknowledges ambiguity and heterogeneity of demonstrations is sketched in the paper. It is argued that it has a rich explanatory and descriptive potential.

*Keywords*: Demonstratives; demonstrations; demonstrate; action–product distinction; referential intentions.

*To Magdalena and two wonderful cat-beings: Boris and Lalunia.*

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## 1. Two profiles of demonstrations

Jack's utterance of

[1] This green is well balanced between blue and yellow

might be accompanied by several types of pointing actions: he might point at this particular patch of green with a finger; he might use a paintbrush to paint a particular patch of green; he might grasp a particular greenish object and show it to the audience; or he might refrain from any overt action if he believes that the particular shade of green is salient in the context. It might even be claimed (although I won't be defending this view here) that the use of "green" in "this green" is nothing more than a constituent of each of the aforementioned pointing actions. In all these cases, the actions in question play a role of a demonstration, and it seems that the following is true of exophoric uses of demonstratives:

(Heterogeneity Thesis) Demonstrations accompanying uses of demonstratives vary with respect to their form[1].

If *heterogeneity* is true (and it is hardly controversial that it is), then one might ask what unifies varieties of possible actions making them exemplifications of demonstrations. One possible answer to this question is provided by the dual-intention model of demonstrations (Ciecierski & Makowski (2022)) according to which all demonstrations – as occurring in acts of communication[2] – are complex actions that have both an *ostensive* and an *intentional* profile.

An ostensive profile of demonstrations comprises any basic behavior that constitutes a demonstration: motor activity of a particular kind (grasping something, pointing with a finger, eye gaze, etc.) or refraining from an overt action. In order to distinguish such basic behaviors from complex acts of demonstration (the former are constituents of the latter), we might refer to

[1] Or, if one prefers not to use the concept of a form: different actions might accompany uses of demonstratives and play a role of a demonstration.

[2] This restriction is important as demonstratives might be employed also in speech acts that are audienceless (cf. Davis (2002)). The model described above does not apply to such cases.

them as "indications" or "demonstrations *sensu stricto*" ("demonstrations$_S$" for short), reserving the terms such as "demonstrations in the broad sense" and "demonstration$_L$" for complex acts of demonstration. *Heterogeneity* assumes, among other things, that the form of a demonstration$_L$ is inherited from the form of an indication that is its constituent, i.e., that if two indications have different forms, the forms differentiate also between demonstrations$_L$ that contain them as constituents. An intentional profile of demonstrations comprises speaker's intentions that accompany particular indications.

In the case of particular demonstrations, of course, the two profiles co-occur, and it is not always easy to tell them apart. However, it should always be in principle possible to single out the ostensive profile by considering, firstly, alternative ostensive interpretations the demonstrations might receive and, secondly, the non-ostensive interpretations it might get. As noted by Wittgenstein (1953: 75), it is possible that a person "naturally reacted to the gesture of pointing with the hand by looking in the direction of the line from finger-tip to wrist, not from wrist to finger-tip"; it is also possible to treat the gesture in the manner characteristic of some animals, as not involving ostension at all.

The aim of this paper is to develop a theory of the ostensive profile of demonstrations in the broad sense and supplement it with a pragmasemantical theory of demonstrative utterances. The theory I shall propose makes use of the action-product distinction. I start with a description of the distinction in question and relate it to intentional profile of demonstrations in the broad sense. Next I describe various truth-conditional ways in which the theory might be developed. Each such way presupposes the idea of pragmatic filter: a manner of determining the class of potential demonstrata. This concept is described in the next section of the paper. The resulting theory has rich descriptive and explanatory potential for dealing with various scenarios of demonstrative communication. It also provides a conceptual framework that enables representing various rival theories of demonstrative utterances.

## 2. The intentional profile of demonstrations

There are at least two dimensions within which one might consider the specific occurrence of an utterance containing a demonstrative and an accompanying demonstration. The first is the communicative dimension: leaving aside the unusual cases of self-directed speech, both the demonstrative utterance and the accompanying gesture must be interpretable. The second is the pragmasemantic dimension: the interpretation that the recipient arrives at should provide an identification of the object of the indication (demonstratum) and the related reference of the demonstrative expression (potentially identical to the demonstratum).

The intentional profile of demonstrations, therefore, comprises at least two aspects: an intention to get the interpreter to form a particular hypothesis regarding the reasons why a particular demonstrations is performed by the speaker (we might call it "abductive intention") and the intention to get the interpreter to form, on the basis of this hypothesis, another one regarding the demonstrated object (we might call it "deictic intention"). Both intentions are, just like indications, constituents of every demonstration in the broad sense.

To illustrate the idea: if an utterance of [1] is accompanied by an act of painting a particular patch of green, the act of using the paintbrush in a certain manner is a case of an indication, while the two accompanying intentions are, respectively, the intention to get the interpreter to form a hypothesis that the speaker used the paintbrush in this particular manner in order to single out a particular shade of green and the intention to get the interpreter to form the hypothesis that this particular shade of green is the demonstratum.

The abductive intention aims at a hypothesis *explaining* the ostetnsive action or, more precisely, aims at making the interpreter to form the hypothesis explaining the ostensive action. The deictic intention aims at attributing *demonstratum* to demonstration (and related reference to a demonstrative). In order to be a possible subject of both roles we need a concept of demonstration that is capable of playing both roles, i.e. one that makes demonstrations a subject-matter of explanation (as actions) and one that makes them subjects of properties such as reference (or their analogues in the case of demonstrations).

## 3. The ostensive profile of demonstrations: the action–product distinction

Uttering [1] is an action that contains other actions (like the action of uttering particular words that occur in [1]) as constituents. However, some words that are used in the utterance are not purely linguistic devices – they are rather *hybrid expressions*, i.e. expressions that contain (to use Frege's well-known formulation) means of expressing the content as parts, constituents or aspects (for various interpretations of this idea see: Künne (1992), (2010), Textor (2007), (2015), Kripke (2008), Penco (2013), Ciecierski (2019)). In case of demonstrative words the respective means are demonstrations. It follows that the action of uttering a demonstrative sentence contains as a constituent the action of using a hybrid expression which contains as a constituent a linguistic expression (i.e. a demonstrative word) and a demonstration.

The concept of *demonstration*s, however, exhibits action–product ambiguity, analogous to that visible in the opposition between jumping and the resulting jump, talking and the resulting talk or crying and the resulting cry. As I shall suggest below, the ambiguity might be linked to two types of intentions that constitute the intentional profile of demonstrations, i.e. corresponds to two aforementioned roles demonstrations has to play.

The action–product distinction was introduced by Kasimir Twardowski in 1911 in his seminal paper *Actions and products: Comments on the broader area of psychology, grammar, and logic*. One of Twardowski's main motivations for introducing the distinction was the rejection of psychologism; however, the distinction is philosophically interesting independently of that motivation. In recent years, for instance, it gained some importance in discussions regarding propositions and propositional attitudes (cf. Moltmann, 2013). It has also been extensively exploited in praxiology (cf. Kotarbiński, 1965; Makowski, 2017). As we shall see below, another area where it might find an application are the debates about demonstratives and demonstrations.

Twardowski introduces the action–product distinction by appealing to the difference in the verb–nouns pairs such as:

to jump – the jump

to shout – the shout

to lie – the lie

to judge – the judgment

to think – the thought

to speak – the speech

to cry – the cry.

As he observes:

> (...) the relation of the verb to its corresponding noun (…) expresses the relation of some action to what emerges as a result of it, owing to, by means of, that action. When we fight, a fight results; when we think, thoughts arise; when we [issue a] command, a command occurs; when we sing, a song results. (Twardowski, 1911: 14-15)

He dubs "that which arises (…) by means of that action" – "the product" of that action (*ibidem*). Hence, the jump is the product of jumping, the shout is the product of shouting, the lie is the product of lying, the judgment is the product of judging, the thought is the product of thinking, etc. By the same token, we might say that the indication (demonstrations) is the product of indicating (demonstratings).

The action–product distinction, however, is not a simple by-product of the verb–noun distinction. As Twardowski notes, immediately in some cases the nouns themselves suffer from action–product ambiguity:

> (...) there is no question that we also frequently make use of a noun for designating an action, which renders these nouns ambiguous, capable as they are of designating now actions, now their products. In the phrase "to take someone's advice," the term "advice" denotes the product of the activity of advising, but when we say: "It's no use giving you advice," we wish to express the sentiment that the activity of offering advice has met with difficulties. (Twardowski, 1911: 15-16)

Twardowski's main argument for the distinction must be, therefore, independent of the linguistic motivations that are behind it. And this is indeed the case: the ground for the distinction is that actions and products have different properties, although in some cases, as Twardowski stresses, it might be difficult to clearly separate a particular action from its product.[3] For instance, the plan but not the action of planning might be implemented, actions, in contrast to some (but not all) products, do not have fulfillment or success conditions (Gerner, 2017: 325). Moreover, all "enduring" (in Twardowski's terminology) products differ with respect to their temporal extension from the corresponding actions (compare: painting qua action and the particular painting that results from it). Some authors claim also the sameness relation might connect products but not actions (Gerner, 2017: 326), as it makes no sense to talk about Jill's jumping being identical to Kate's, while we might truly say that Jill's jump was identical to Kate's. However, this last observation is very problematic as it seems to be based on the confusion of types with tokens: my thought as a product might be identical with yours if we talk about the type while my action of thinking is different from yours if we pay attention to two actions-tokens. If we, however, compare tokens to tokens and types to types the sameness relation seems to be equally applicable or inapplicable to the respective action-product pairs.

It is important not to confuse products of actions and arbitrary effects of actions. Although every product is an effect of some action, not every effect of an action counts as its product. Producing a particular vowel is an effect of talking but only the entire talk counts as the product of talking. The criterion that enables distinguishing arbitrary effects from products is intentional: the product is the intended effect of a *whole* action that is, at the very same time, constituted and necessary determined by the action as

[3] As Brandl (1998) notes: there are at least two possible interpretations of Twardowski's considerations. The first requires a categorial ontological difference between actions and products according to which actions and products constitute inseparable wholes but might be nevertheless distinguished conceptually as distinct entities. The second requires a difference in meaning without a difference in reference and ontology. Here I am assuming (contrary to Brandl's suggestions) the correctness of the first interpretation.

a whole. I might, for instance, talk in order to achieve a certain persuasive goal but it will not count as the product of my action of talking because – even if it is intended as the effect of the entire action – it is not constituted or necessary determined by it: nothing in the talk itself secures the effect in question. This contrasts clearly with the case of the talk as a whole which is constituted by the action in question[4].

Following (and slightly modifying) the suggestion of Brandl (1998), we might represent the ambiguity in terms of Davidsonian event-semantics. The sentence:

[2] Magdalena shouted at Boris.

might be interpreted as (action-directed reading)[5]

[2A] $\exists e \exists t$ [Shouting($e$) $\wedge$ Agent(Magdalena, $e$) $\wedge$ Patient(Boris, $e$) $\wedge$ Time(t, $e$) $\wedge$ t $<$ $\mathbf{t}_0$]

while the sentence 'Magdalena's shout at Boris was loud' (product-directed reading) either as:

[2Pn] $\exists e \exists t \exists x$ [Shouting($e$) $\wedge$ Agent(Magdalena, $e$) $\wedge$ Patient(Boris, $e$) $\wedge$ Time(t, $e$) $\wedge$ t $<$ $\mathbf{t}_0$ $\wedge$ Product ($x$, $e$) $\wedge$ Shout($x$) $\wedge$ Loud($x$)] [6].

or as:

[2Pe] $\exists e \exists e' \exists t \exists x$ [Shouting($e$) $\wedge$ Agent(Magdalena, $e$) $\wedge$ Patient(Boris, $e$) $\wedge$ Time(t, $e$) $\wedge$ t $<$ $\mathbf{t}_0$ $\wedge$ Product ($e'$, $e$) $\wedge$ Shout($e'$) $\wedge$ Loud($e'$)]

depending on how we would like to treat products in our ontology: as events (2Pe) or entities of (potentially) some other category (2Pn).

Similar differences can be found in the case of indications qua actions and indications qua products. The former might be a subject matter of

[4] Let us note, however, that the idea of nonendring products (e.g. jump as a product of jumping or demonstration as a product of demonstrating) has been recently criticized by some authors (cf. Bronzo (2020)). I am not offering here a reply to this criticism as the issue deserves an independent study.

[5] $\mathbf{t}_0$ represent here the time of utterance.

[6] I leave open the question of whether $e$ and $x$ range over a single category of entities (events).

psychological explanation ("Why she behaved like this, i.e., why she performed this particular act of indication?"), while nothing similar applies to the latter (the question "Why did the particular indication qua product occurred?" is not the question about the psychological factors responsible for the occurrence of a particular event).

Additional support for the applicability of the distinction to cases of indications comes from modal considerations. Consider, for instance, the following scenario (de Gaynesford, 2008: 169):

> [Scenario 1] The speaker points with a finger towards a horse (A) but another horse (B) replaces A during the utterance of "that's my horse" when the speaker closes her eyes for a second.

And contrast it with the following one:

> [Scenario 2] The speaker points with a finger towards a horse (A) during the utterance of "that's my horse". She closes her eyes for a second but no other horse replaces A during pointing.

In the first case, a certain demonstration qua product (DP1) and a certain demonstrations qua action (DA1) co-occur, while in the second scenario the very same demonstration qua action (DA1) is accompanied by a different demonstration qua product (DP2). At least in some cases, therefore, a demonstration qua action might co-occur with a distinct demonstration qua product.

If we agree that the distinction is well-founded, we are entitled to claim that[7]:

[3] Jill's demonstrations accompanying the utterance of 'this'$^{<l,t>}$ is vague.

might receive the following two readings:

[3A] $\exists e$[Indicating($e$) $\wedge$ Agent(Jill, $e$) $\wedge$ Time(**t**, $e$) $\wedge$ Utters(Jill, "*this*"$^{<l,\ t>}$, **t**) $\wedge$ Vague($e$)]

[7] Following the idea of Reichenbach and others (cf. Ciecierski (2020)) we are using token quotes "$x$"$^{<l,\ t>}$ that refer to a particular token of an expression $x$ having a particular spatiotemporal characteristics marked as <l, t>.

[3Pn] $\exists e \exists x$[Indicating($e$) $\wedge$ Agent(Jill, $e$) Time(**t**, $e$) $\wedge$ Utters(Jill, "*this*"$^{<l, t>}$, **t**) $\wedge$ Indication($x$) $\wedge$ Product($x$, $e$) $\wedge$ Vague($x$)][8]

corresponding, respectively, to action-directed reading of [3] and product-directed reading of [3][9].

As we have observed above products – in contrast to actions – might have success or fulfilment conditions. However, this does not mean that every product has them. Compare, for instance, expectation and jump. The former can be fulfilled as it makes sense to say of a certain expectation that it concerns a certain state of affairs and that the state of affairs in question occurred or not. At the very same time nothing similar can be said of jump. Is demonstration$_s$ the product of the first or of the second type? Consider again the two horse racing scenarios described above. In both cases it is clear that we might attribute to the speaker several intentions including the one regarding the correct hypothesis to be guessed by the interpreter. Now, in the first scenario the interpreter or rather the *rational* interpreter (the actual but deluded or inattentive cannot be proxy for the success of demonstration) will be unable to guess the reasons for performing the demonstration. Hence the demonstration will be unsuccessful. In the second scenario, on the other hand, she will be able to form the correct hypothesis explaining the behavior of the speaker. Hence the demonstration will be successful. This illustrates the sense in which demonstrations qua products have success or fulfilment conditions.

## 4. Demonstrata: potential, intended and actual

In both scenarios the situation was relatively simple. However, it might happen that the interpreter will end up not with one candidate for the explanatory hypothesis but with several ones that are consistent with what is known about the context and the demonstrative behavior and there are

[8] Or: [3Pe] $\exists e \exists e' \exists x$[Indicating($e$) $\wedge$ Agent(Jill, $e$) Time(**t**, $e$) $\wedge$ Utters(Jill, "*this*"$^{<l, t>}$, **t**) $\wedge$ Indication($e'$) $\wedge$ Product($e'$, $e$) $\wedge$ Vague($e'$)], if one wants to treat products as events.

[9] Action-product ambiguity applies here also to the notion of utterance – I am ignoring it for the sake of presentation.

several ways of unpacking the idea of success conditions for demonstrations in such cases. One requires that the demonstration is successful in contributing an object to truth-conditions only if it either has a singular interpretation (the multiplicity of hypotheses is not the case) or if (assuming that the multiplicity of hypotheses holds) among its interpretations there is one which captures the intended demonstratum. Another pays no special attention to cases of singular interpretation and treats cases of mismatch between intended demonstratum and potential demonstrata as resulting in truth-value gaps. I do not have any knockdown argument for or against one of the options (nor against other possible extensions of the framework) – both might be included in the truth-conditional extensions of the theory sketched in this paper (compare: Truth Conditions 1 and 2 given below).

Within the ostensive profile of demonstrations, indications qua products — as having success or fulfillment conditions — contribute *candidates* for the object demonstrated (*potential* demonstrata), while the intentional profile of demonstration contributes the intended demonstratum. Now what is the *actual* demonstratum depends on the relation between the two or rather on theoretical constraints that a semantic theory imposes on the relation in question. Here are some (but definitely: not all possible) ways of developing the idea.

The first looks as follows. If the intended demonstratum is on the list of potential demonstrata, then it is the actual demonstratum. If it is not, then, depending on how big the class of potential demonstrata is, there is no actual demonstratum or the demonstrarum is the only object that is the potential demonstratum (in cases where demonstrations contributes a single object). More formally: let $c$ be a context that contains $s$ as the speaker, $i$ as the indication qua product, $D_i$ as the class of potential demonstrata that correspond to $i$, and $D_s$ as the (singleton) class whose only element is the individual the speaker has in mind. For the utterance $u$ of "This is F," the corresponding truth conditional clause takes the following form:

(TRUTH CONDITIONS 1)

$u$ is true in $c$ that contains $s$, $i$, $D_i$ and $D_s$ iff (i) every $x$ in $D_i \cap D_s$ is F and $D_i \cap D_s \neq \varnothing$ or (ii) every $x$ in $D_i$ is F and $D_i \cap D_s = \varnothing$ and $|D_i| = 1$.

$u$ is false in $c$ that contains $s$, $i$, $D_i$ and $D_s$ iff (i) every $x$ in $D_i \cap D_s$ is not F and $D_i \cap D_s \neq \varnothing$ or (ii) every $x$ in $D_i$ is not F and $D_i \cap D_s = \varnothing$ and $|D_i| = 1$.

$u$ lacks truth value in $c$ that contains $s$, $i$, $D_i$ and $D_s$ iff $D_i \cap D_s = \varnothing$ and $|D_i| > 1$.

This analysis follows the intuition of those who believe that demonstration and intention are jointly decisive for demonstrative reference but who also claim that in some cases (when there is only one potential demonstratum) demonstration might take over and become a decisive factor. This interpretation might be treated as a version of Kaplan's account from *Dthat* (Kaplan (1978)) which stresses the importance of demonstrations and contextual cues while attributing purely disambiguating role to referential intentions.

Here is another way in which we might develop the idea: if the intended demonstratum is on the list of potential demonstrata, then it is the actual demonstratum. If it is not, then there is no actual demonstratum. Here, the corresponding truth conditional clause takes the following form:

(TRUTH CONDITIONS 2)

$u$ is true in $c$ that contains $s$, $i$, $D_i$ and $D_s$ iff every $x$ in $D_i \cap D_s$ is F and $D_i \cap D_s \neq \varnothing$.

$u$ is false in $c$ that contains $s$, $i$, $D_i$ and $D_s$ iff every $x$ in $D_i \cap D_s$ is not F and $D_i \cap D_s \neq \varnothing$.

$u$ lacks truth value in $c$ that contains $s$, $i$, $D_i$ and $D_s$ iff $D_i \cap D_s = \varnothing$.

This analysis follows the idea that a speaker's intentions determine the reference of a demonstrative, but only if he or she selects one of the potential demonstrata. It also assumes the thesis (cf. Roberts (1997): 191) that demonstrations do not override the referential intentions.

The two options are not the only available. We might, for instance, spell out a view (also considered as an option in *Dthat* but not supported by Kaplan himself) one might call *strong demonstrativism* according to which the only thing that truth-conditionally matters is the class of potential demonstrata:

(Strong demonstrativism)

$u$ is true in $c$ that contains $s$ and $D_i$ iff every $x$ in $D_i$ is F and $|D_i| = 1$.

$u$ is false in $c$ that contains $D_i$ iff every $x$ in $D_i$ is not F and $|D_i| = 1$.

$u$ lacks truth value in $c$ that contains $s$ and $D_i$ iff $D_i = \varnothing$ or $|D_i| > 1$.

Which stands in a direct opposition to *strong intentionalism* (the view of Kaplan from *Afterthoughts* and other intentionalists like Radulescu (2019)) which claims that the referenatial intentions are the only thing that matters:

(Strong intentionalism)

$u$ is true in $c$ that contains $s$, $i$, $D_S$ iff every $x$ in $D_S$ is F.

$u$ is false in $c$ that contains $s$, $i$, $D_S$ iff every $x$ in $D_S$ is not F.

From the viewpoint of strong intentionalism and strong demonstrativism theories that embrace (TRUTH CONDITIONS 1) or (TRUTH CONDITIONS 2) are hybrid views that combine intentionalism and demonstrativism[10].

An orthogonal with respect to the previous extensions is the one that assumes a dependence of the intended demonstratum on the fact that a particular object counts an the unique potential demonstratum. It is orthogonal as it provides an answer to the question how the intended demonstratum is determined. The truth-conditional clause it makes would be analogous to that of strong intentionalism but it could not be treated as a version of strong intentionalism due to the fact that the determnination of the intended demonstratum is not purely subjective,

Last but not least, the truth conditional analyses presented above are deliberatively simplified as the actual demonstratum (if there is one) does not have to be the *referent* of the corresponding demonstrative. In regular cases it has this status, but in the cases of deferred reference such as:

[10] For a discussion regarding the role of intentions and demonstrations in truth conditional interpretation of demonstrative utterances, see Reimer (1991), Bach (1992), Roberts (1997), Perry (2009), King (2014), Radulescu (2019), and Leth (2020).

[4] This [the speaker shows a copy of *Promise me, dad*] is the current president of the USA[11]

the relation between the actual demonstratum (the copy of the book) and the referent of demonstrative (Joe Biden) is indirect (cf. Nunberg, 1993). So the truth conditional clauses should also include the relation of representation that holds between the demonstratum and the demonstrated object.

The choice of a particular version of truth conditional theory depends on additional philosophical arguments and motivations that I shall not offer in this paper.

With the exception of strong intentionalism, all the analyses presented above make some use of the concept of potential demonstratum. The rough intuition is that:

> Demonstrations$_S$ qua products along with some presuppositions regarding the relevance of particular factors determine potential demonstrata.

Consider, for instance, the following scenario (a modified version of the example discussed by Reimer, 1991):

> Suppose that Peter grabs a bunch of keys from his desk while saying "These are mine". The bunch actually contains some keys that are Peter's and some that are not.

Here the list of potential demonstrata comprises all the sub-collections of keys from the bunch grasped by Peter. The relevant factors concern the presuppositions regarding the rationale behind Peter's behavior.

Or consider the following scenario:

> I am sitting on Venice beach on a crowded holiday looking south, with swarms of people in sight. I fix my attention on a woman in the distance, and, intending to talk about her and gesturing vaguely to the south, say "She is athletic". (King, 2014: 224)

Here the list of potential demonstrata comprises all the females visible within the scope of the vague gesture. The relevant factors, again, concern

[11] Following Kaplan (1989), the description within the brackets is a description of a demonstration, i.e., it is not a part of what is said.

the presuppositions regarding the rationale behind the speaker's behavior. Such a presupposition determines that we are talking about persons visible within the scope of the gesture who have a certain gender.

Consider, finally, a scenario inspired by one of John Perry's (1997) examples:

> Someone utters the sentence of the language EL*: "That fish was yea big," which differs from English only in that EL* contains the expression "yea," which conventionally always refers to the distance between the hands of the speaker. While uttering the expression, the speaker is making a suitable gesture.

Here the list of potential demonstrata contains a single element being a particular length. The relevant factor here is that we are employing a certain (strict) linguistic convention that precisely determines the relation between the gesture and the object demonstrated.

## 5. Pragmatic filter

Let us call the mechanism of employing certain factors in the process of the determination of potential demonstrata a *pragmatic filter*. There are, I think, at least two ways in which one may attempt to explicate this concept.

The first one appeals to the already introduced idea of the rational interpreter of a demonstrations. According to that approach, potential demonstrata are the objects a rational interpreter might consider as demonstrata when forming the hypothesis explaining the act of indicating. Sometimes there are many hypotheses at stake, and the approach predicts that the class of potential demonstrata becomes numerous. A rational interpreter, as one might assume, knows the context of an utterance well, including expectations and background assumptions shared by the *actual* participants at a given stage of the conversations, but excluding the knowledge of those of the speaker's attitudes and intentions that are not intersubjectively decodable.

For instance, in the key scenario, the most likely reason for grasping the bunch of keys while uttering "these" is to demonstrate at least some (but potentially all) keys from the bunch. This is at least the most likely folk

psychological generalization regarding the action involving grasping this or that bunch of keys. This is even more transparent if the previous conversation concerned the speaker's plan to return home or the if end of the work hour is approaching. But this might be canceled given alternative constraints imposed by the context. In the Venice-beach scenario, the most likely reason for using the pronoun "she" and making the gesture have a certain direction and scope is to single out a person located in that direction within that scope and (at least) looking as having a particular gender. Given that assumption, the candidate for a demonstratum is every object that satisfies the general constraints. Finally, in the fish scenario, the crucial assumption regarding the context is that the speaker is exploiting a certain convention linking "yea" with a certain abstract object being the length.

Consider yet another scenario. Suppose that in a certain building there are two rooms that are phenomenally nearly indiscernible. One contains the portrait of Carnap, the other – the portrait of Agnew. The speaker mistakes the second room for the first and without looking at the wall utters: "This is a picture of one of the greatest philosophers of the twentieth century". She thinks that she is in the room that contains the picture of Carnap but is actually in the room that contains the picture of Agnew. In this case, the normal, attentive and reasonable participant of the conversation will not be able to guess that the mistake has been made so she will consider the portrait of Agnew as the only object that is the candidate for the demonstratum. However, if the mistake is common and it is an element of the background knowledge that it is easy to mistake the rooms, the situation changes dramatically: both portraits may become candidates for a potential demonstratum in such cases.

The theory of demonstrative utterances that is closest to this interpretation of the idea of pragmatic filter is the coordination account of Jeffrey King (2014). Its main semantic point is that the referent of the demonstrative in the context must meet two conditions: (A) it must be intended as a referent by the speaker, and (B) "a competent, attentive, reasonable hearer" must recognize it as the intended referent (ibidem, 225). As far as "a competent, attentive, reasonable hearer" means "the rational interpreter," the accounts share the common intuition that the speaker must do enough to enable the recognition of the intended object in the context. They differ,

however, with respect to the assumption of what counts as "enough": in King's account, a single object must be recognizable, while in the account sketched in this paper, this applies to potentially numerous classes of objects. Another difference between the accounts is that King talks about the intended referent of the demonstrative, while the account described here talks about the intended demonstratum. The difference might not be visible in regular cases, but in cases involving deferred reference, the two objects might be different. Additionally, the predictions of King's account and the account sketched in this paper might differ in particular cases. For instance, if there is only one female-looking object within the scope of the gesture (King's original scenario does not specify this), the interpretation sketched above predicts that the reference is secured no matter what the truth-conditional extension of the account looks like. The predictions of King's account depend here on whether a competent, attentive, reasonable hearer is capable of singling out the object which must be (at the very same time) intended as the referent. In cases where the number of objects that count as female-looking is greater than one, the prediction regarding reference depends on the choice of a particular truth-conditional extension of the theory: in the case of weak demonstrativism, for instance, the reference is secured as far as the deictic intention of the speaker matches at least one of the female-looking objects; in the case of strong demonstrativism, the reference is not secured. Here we may actually apply King's idea and treat conditions he proposes as additional constraints that take us from the class of potential demonstrata into the the actual demonstratum (and referent, if we are not dealing with the case of deferred reference)[12].

The alternative method of unpacking the idea of pragmatic filter is to appeal to Kaplan's (1989) idea of the Fregean Theory of Demonstrations (FTD) but slightly modify it to enable situations in which the "reference" of a demonstration is not singular and apply it outside of the domain of perceptual demonstratives (the restriction assumed by Kaplan). Kaplan (1989) suggested (he abandoned the theory later) that demonstrations can be adequately characterized in terms of the (appropriately extended) Fregean categories of manner of presentation and reference:

[12] I would like to thank one of the anonymous reviewers of this paper for bringing this to my attention.

> (...) the analogy between descriptions (...) and demonstrations is close enough to provide a sense and denotation analysis of the <<meaning>> of a demonstration. The denotation is the demonstratum (...), and it seems quite natural to regard each demonstration as presenting its demonstratum in a particular manner, which we may regard as the sense of the demonstration. The same individual could be demonstrated by demonstrations so difference in manner of presentation that it would be informative to a competent auditor-observer to be told that demonstrata were one. (514)

Kaplan discusses several principles that govern the use of demonstrations and enable to establish "isomorphism" between demonstrations and definite descriptions. The most important are (the names of the principles are mine):

**The Basic Principle**
"A demonstration is a way of presenting an individual" (Kaplan, 1989: 525)

**The Principle of Non-rigidity**
"It is not required that an occurrence of a demonstration have a fixed content." (Kaplan, 1989: 525)

**The Principle of Contingent (non-)Emptiness**
"A demonstration which fails to demonstrate any individual might have demonstrated one, and a demonstration which demonstrates an individual might have demonstrated no individual at all." (Kaplan, 1989: 525)

**The Detachment Principle**
"A given demonstration might have been mounted by someone other than its actual agent, and might be repeated in the same and different place." (Kaplan, 1989: 525)

**The Involvement Principle**
"(...) it does seem to me to be essential to a demonstration that it presents its demonstrata from some perspective, that is, as the individual that looks thusly from here now." (Kaplan, 1989: 525)

Some comments concerning the principles are in order. The *Basic Principle* equates demonstrations with ways of presenting demonstrata. It has an easily identifiable analogue in the realm of descriptions: they can also be said to be ways of presenting the things described, ways that exploit properties expressed by the appropriate predicates occurring in the description. It entails, among other things, that the notion of a demonstrating procedure becomes very capacious: it can be applied to all appropriately situated manners of presenting an individual. This consequence is very welcome: the diversity of possible ways of demonstrating something is an empirical fact that must be somehow acknowledged by every adequate theory of demonstratives and demonstrating procedures. FTD offers exactly this: a flexible notion of demonstration and the support for *heterogeneity*.

The *Principle of Non-rigidity* and the *Principle of Contingent (non-)Emptiness* state together that demonstrating procedures might behave like non-rigid definite descriptions that are neither necessarily empty nor necessarily non-empty. This does not, however, rule out cases of rigid demonstration. In fact, our "yea" example belongs precisely to this category (the respective convention warrants that the connection between the distance and the length is fixed across all possible worlds). The *Detachment Principle* and the *Involvement Principle* attempt to draw a demarcation line between essential and contingent properties of demonstrating procedures; the latter attempts also to provide identity conditions for them. Although the *Detachment Principle* says that the location of a demonstration is not essential, while the *Involvement Principle* says that the perspective (which is essential for the demonstrating procedure) involves somehow both time (as here) and place (as now), there is no inconsistency here: the values of here and now are supplemented contextually (Kaplan writes here about setting a demonstration in a context) and, though determinative for the perspective, are external with respect to it. The *Involvement Principle* applies only to visual demonstrating procedures and uses of perceptual demonstratives, but I see no reason why it could not be extended to other kinds of demonstrating procedures. This would require, of course, a capacious enough idea of "looking thusly" from a certain perspective as well as a detailed analysis of various roles that senses or manners of presentations might play (cf. Zalta, 1988: 154-158). Last but not least, as I have stressed

above, we also have to consider another modification of the theory. In order to accommodate the idea of the class of potential demonstrata being numerous, demonstrations should be interpreted here as analogous to indefinite descriptions. This does not require substantial changes in principles governing FTD.

The two interpretations of the idea of pragmatic filter differ with respect to the way they approach the problem of determination of potential demonstrata. Roughly speaking, the interpretation appealing to the idea of a rational interpreter claims that:

(A) An indication *I* qua product contributes $a_1...a_n$ as potential demonstrata in virtue of the link among $a_1...a_n$, the beliefs of the rational interpreter *R*, and an indication qua action that has *I* as its product.

while the FTD-motivated interpretation holds:

(B) An indication *I* qua product contributes $a_1...a_n$ as potential demonstrata in virtue of the link between $a_1...a_n$ and the properties $F_1...F_n$ of the indication qua action that has *I* as its product.

It should be noted here, however, that the two characteristics are not logically exclusive. In particular, one may want to ask: "what grounds the link between particular properties of an indication and potential demonstrata?". And (A) might provide an answer to this question supplementing the 'semantics' of indications (B) with the appropriate "metasemantics." According to such a hybrid approach, the relevant properties qua being responsible for the fact that an indication is associated with a certain manner of presentation have this status because without the properties in question, it would be difficult (potentially impossible) to explain or to make sense of the occurrence of the indication in this particular context.

Indications might be also more or less conventionalized. Roughly speaking, the more conventionalized an indication is more explicit are the properties an object must posses in order to count as a potential demonstratum. Less conventionalized an indication is less explicit and more current knowledge dependent the choice of relevant properties is (we might think of the typology of indications resulting from a degree of its

conventionaliation in terms of the *ostensive continuum*). Another way of classifying (A) and (B) is to think of the former as applicable to conventionalized indications and to think of the latter as applicable to non-conventionalized ones.

I do not want know now to decide which of the two interpretations is more accurate. My aim here is modest: I want stress that that the two accounts of pragmatic filter are supplementary rather than contradictory.

Let me close this section by showing how the ideas of pragmatic filter and demonstration qua product might be employed in the analysis of more complex examples. Consider the following scenario (Siegel (2002), Radulescu (2019)):

> You are a salesman in a tie store. By reaching past an opaque door into a display case, you put your hand on a blue silk tie. At the same time, another salesman is reaching through the cabinet and touching a red silk tie. Through the glass top of the cabinet, you can see the red tie being held by the other salesman, whose arm looks like yours. You mistake his hand for yours and you believe that you are the one touching the red tie. You say to a customer, who was looking in another direction for a red silk tie, 'This one is red'.

The theory presented here gives us the opportunity to provide an account of the scenario as involving an *ambiguous utterance* (I believe also that such an analysis is intuitively compelling). In particular, we might note that there are two indications that occur in the scenario: one connected with gaze and the other with touch. Given this we actually have the following two distinct speech acts that are packed in the single utterance:

[i1] <'This one', indication$_1$> (where: indication$_1$ = the gaze)

[i2] <'This one', indication$_2$> (where: indication$_2$ = touching)

The pragmatic filter in the first case indicates objects towards which the gaze is directed and which meet some additional circumstance-sensitive conditions connected with practical interests of the participants of the exchange (e.g. that presupposition that the conversation concerns ties etc.). The second speech act involves objects that are touched or are parts of an object that is touched that also meet some additional circumstance-sensitive

conditions. The two indications contribute two distinct (singleton) classes as sets of potential demonstrata neither of which deserves to be called the actual class of potential demonstrata. The appeal to plans of the speaker and to intentions in general is irrelevant when addressing the question what proposition has been expressed by the utterance. At the very same time it is relevant (as it should be) when addressing the question regarding the charitable interpretation of the utterance and the resulting disambiguation.

## 6. Conclusion

Considerations presented in this paper show, firstly, that demonstration$_s$ qua actions and demonstrations$_s$ qua products might be conceived as linked respectively to two dimensions of every demonstrative utterance: the intention to ask the interpreter for an explanation of the action of pointing and the intention to make her guess the intended demonstratum. Secondly, they show that the theory that comprises the distinction can be truth-conditionally developed in several ways, making it compatible with selected assumptions of demonstrativism and intentionalism. Finally, they also show the need of an additional theory explaining how the class of potential demonstrata is determined. As I have suggested above, the explanation might make use of the idea of a rational interpreter of an indication as well as of the Fregean Theory of Demonstrations.

The general picture of demonstrative communication that emerges from the framework presented above puts a special stress on the 'interaction' between the speaker and the rational interpreter. The latter concept plays, firstly, the role in determining the class of potential demonstrata and, secondly, that of a factor that determines the fulfillment or success conditions for demonstrations. One of the main questions that emerges from it is if the analogous considerations could be applied to pure indexicals which, at least according to the popular picture[13], have the reference secured automatically, irrespective of the attitudes of the participants of the conversational situation. I think that, among others, the cases of distributed utterances, that is

[13] The picture has been challenged by several authors (cf. Predellli (2005), Mount (2008)) but remains popular among many others.

cases of utterances where more than one indexical occurring in the sentence is linked to a single aspect of the context which may take different values relevant for the interpretation of the respective indexicals (like in the sentence: 'It is now 3 o'clock and it is now past three' uttered by the speaker who intentionally started speaking at 3 o'clock but finished one past three) suggest a similar pragmasemantical mechanism governing the use of indexicals and demonstratives (cf. Ciecierski (2019)). The mechanism in question takes into the account both the intentions of the speaker and the class of potential values of contextual parameters predicting that the successful reference emerges as the result of interaction between the two factors. If I am correct, the theory sketched in this paper may prove to be an important building block of the unified account of indexicals and demonstratives.

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